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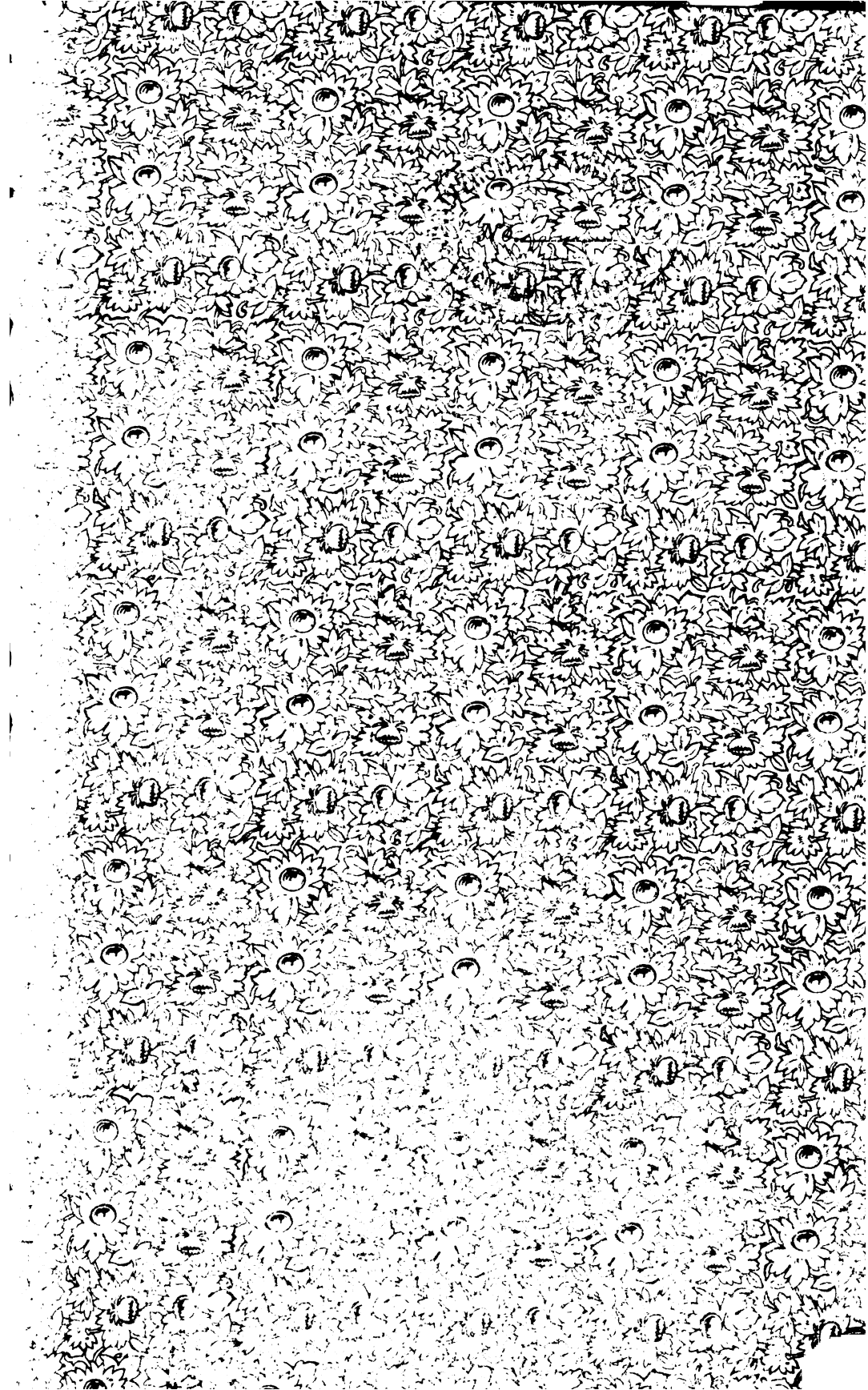
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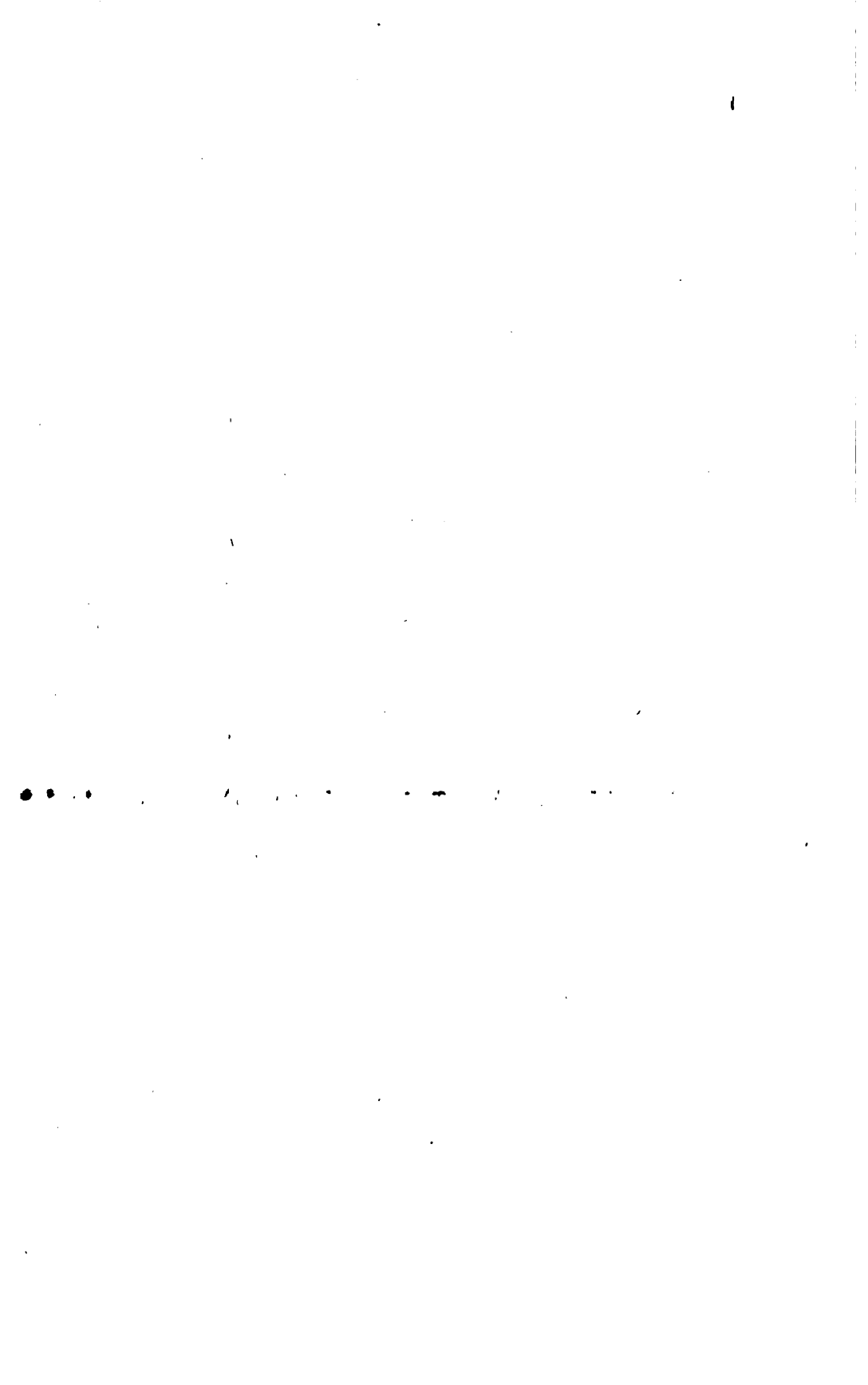
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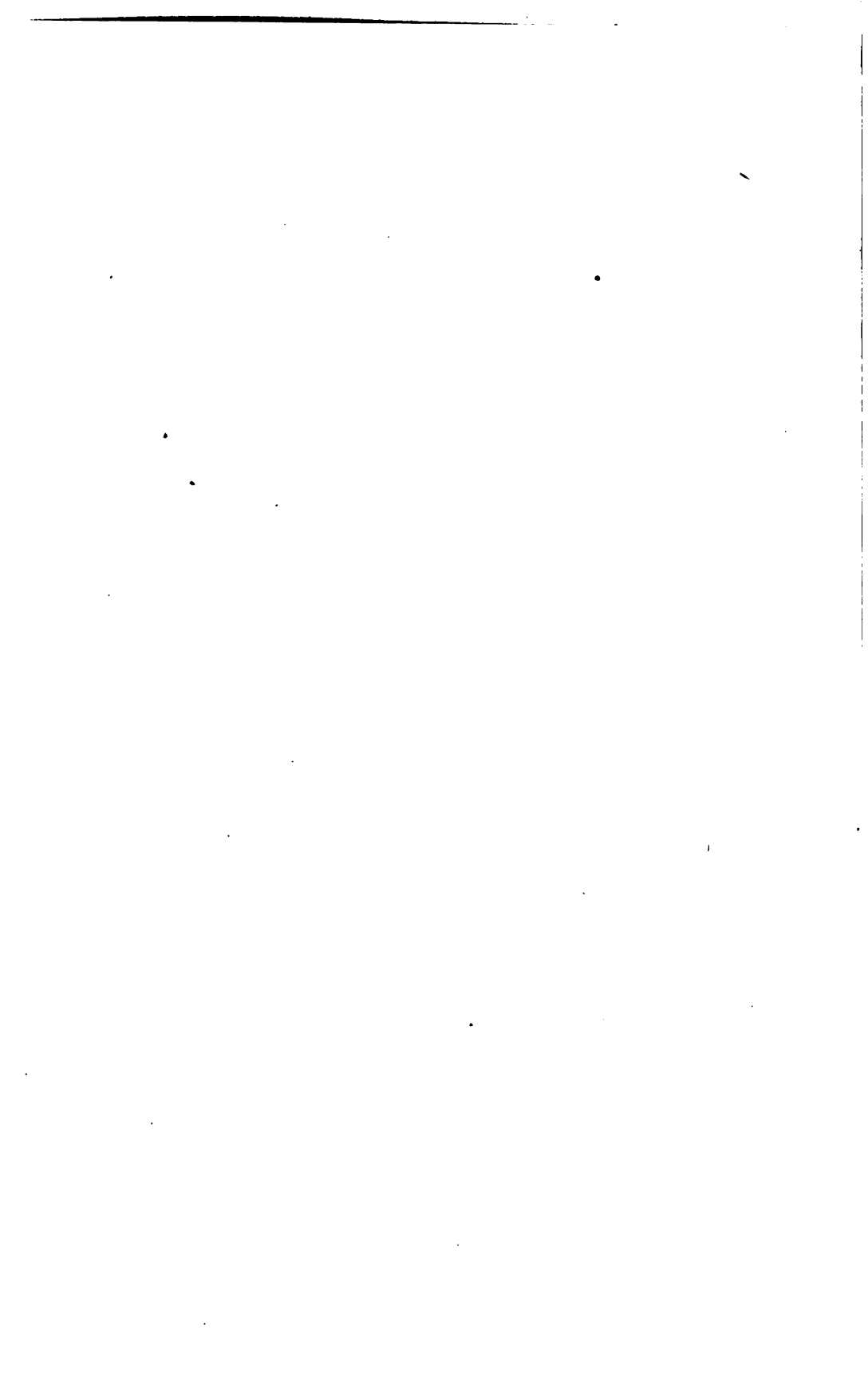






WORKS OF THE  
**BROWN & SHARPE MFG. CO.,**  
Providence, R.I.





PRACTICAL TREATISE  
OF  
GEARING.

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SECOND EDITION.

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## PREFACE.

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This Book is made for men in practical life; for those who would like to know how to construct gear wheels, but whose duties do not afford them sufficient leisure for acquiring a technical knowledge of the subject.

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# PART I.

## CHAPTER I.

### PITCH CIRCLE, PITCH, TOOTH, SPACE, ADDENDUM OR FACE, FLANK, CLEARANCE.

Let two cylinders, Fig. 1, touch each other, their <sup>Original Cyl-</sup>axes be parallel and the cylinders be on shafts, turning <sup>inders.</sup>freely. If, now, we turn one cylinder, the adhesion of its surface to the surface of the other cylinder will make that turn also. The surfaces touching each other, without slipping one upon the other, will evidently move through the same distance in a given <sup>Linear Velocity.</sup>time. This surface speed is called *linear velocity*.

#### TANGENT CYLINDERS.

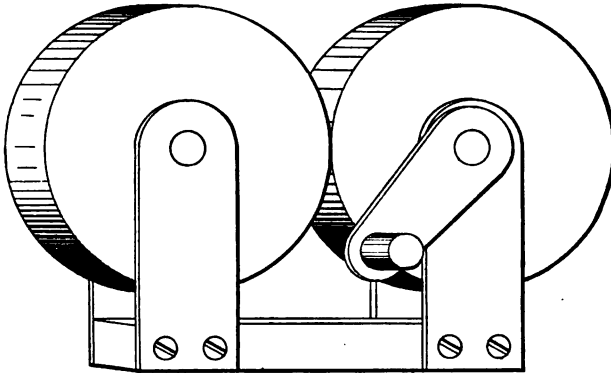


Fig. 1

**LINEAR VELOCITY** is the distance a point moves along a line in a unit of time.

The line described by a point in the circumference of either one of these cylinders, as it rotates, may be called an arc. The length of the arc (which may be greater or less than the circumference of cylinder), described in a unit of time, is the velocity. The length, expressed in linear units, as inches, feet, etc., is the linear velocity.

The length, expressed in angular units, as degrees, is the **angular velocity**.

If now, instead of  $1^\circ$  we take  $360^\circ$ , or one turn, as the angular unit, and 1 minute as the time unit, the angular velocity will be expressed in turns or revolutions per minute.

If these two cylinders are of the same size, one will make the same number of turns in a minute that the other makes. If one cylinder is twice as large as the other, the smaller will make two turns while the larger makes one, but the linear velocity of the surface of each cylinder remains the same.

This combination would be very useful in mechanism if we could be sure that one cylinder would always turn the other without slipping.

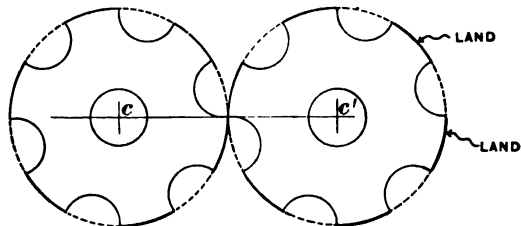


Fig. 2

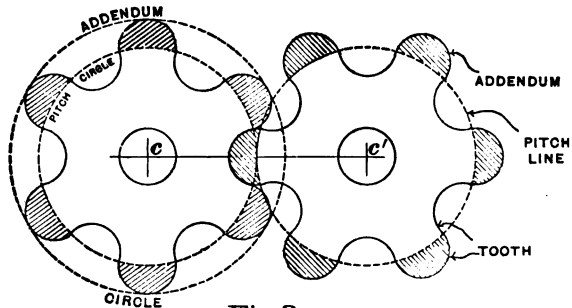


Fig. 3

In the periphery of these two cylinders, as in Fig. 2, cut equidistant grooves. In any grooved piece the places between grooves are called *lands*. Upon the lands add parts; these parts are called *addenda*. A land and its addendum is called a *tooth*. A toothed cylinder is called a *gear*. Two or more gears with teeth interlocking are called a *train*. A line,  $c, c'$ , Fig.

Land.

Addendum.

Tooth.

Gear.

Train.

2 or 3, between the centers of two wheels is called the *line of centers*. A circle just touching the addenda is called the *addendum circle*.

Line of Centers.  
Addendum Circle.

The circumference of the cylinders without teeth is called the *pitch circle*. This circle exists geometrically in every gear and is still called the pitch circle or the primitive circle. In the study of gear wheels, it is the problem to so shape the teeth that the pitch circles will just touch each other without slipping.

Pitch Circle.  
Pitch Circle is also called the Primitive Circle.

On two fixed centers there can turn only two circles, one circle on each center, in a given relative angular velocity and touch each other without slipping.

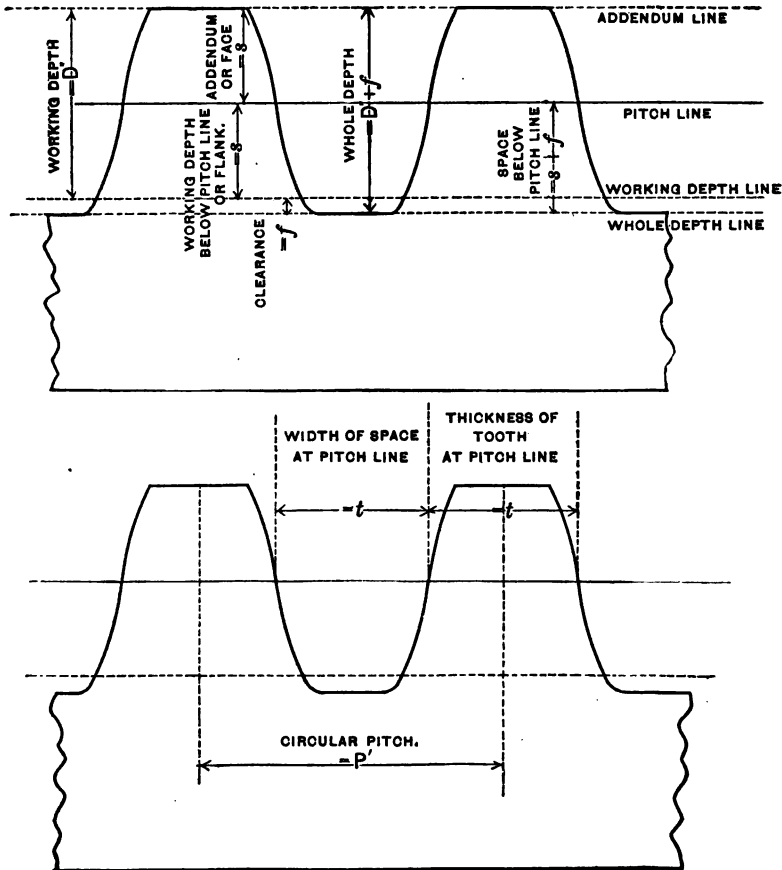


Fig. 4

Space.

The groove between two teeth is called a *space*. In cut gears the width of space at pitch line and thickness of tooth at pitch line are equal. The distance between the center of one tooth and the center of the

Circular Pitch. next tooth, measured along the pitch line, is the *circular pitch*; that is, the circular pitch is equal to a tooth and a space; hence, the thickness of a tooth at the pitch line is equal to one-half the circular pitch.

Tooth Thickness.

Abbreviations of Parts for Teeth and Gear.

Let  $D$  = diameter of addendum circle.

"  $D'$  = " " pitch "

"  $P'$  = circular pitch.

"  $t$  = thickness of tooth at pitch line.

"  $s$  = addendum or face, also length of working part of tooth below pitch line or flank.

"  $2s = D'$  or twice the addendum, equals the working depth of teeth of two gears in mesh.

"  $f$  = clearance or extra depth of space below working depth.

"  $s + f$  = depth of space below pitch line.

"  $D' + f$  = whole depth of space.

"  $N$  = number of teeth in one gear.

"  $\pi = 3.1416$  or the circumference when diameter is 1.

$P'$  is read "P prime."  $D''$  is read "D second."  
 $\pi$  is read, "pi."

If we multiply the diameter of any circle by  $\pi$ , the product will be the circumference of this circle. If we divide the circumference of any circle by  $\pi$ , the quotient will be the diameter of this circle.

To find the Circumference and Diameter of a Circle.

## CHAPTER II.

# CLASSIFICATION—SIZING BLANKS AND TOOTH PARTS FROM CIRCULAR PITCH—CENTER DISTANCE—PATTERN GEARS.

If we conceive the pitch of a pair of gears to be <sup>Elements of</sup> the Teeth. made the smallest possible, we ultimately come to the conception of teeth that are merely lines upon the original pitch surfaces. These lines are called *elements* of the teeth. Gears may be classified with reference to the elements of their teeth, and also with reference to the relative position of their axes or shafts. In most gears the elements of teeth are either straight lines or helices (screw-like lines).

In PART I. of this work, we shall treat upon THREE KINDS OF GEARS:

*First*—SPUR GEARS; those connecting parallel shafts <sup>Spur Gears.</sup> and whose tooth elements are straight.

*Second*—BEVEL GEARS; those connecting shafts <sup>Bevel Gears.</sup> whose axes meet when sufficiently prolonged, and the elements of whose teeth are straight lines. In bevel gears the surfaces that touch each other, without slipping, are upon cones or parts of cones whose apexes are at the same point where axes of shafts meet.

*Third*—SCREW OR WORM GEARS; those connecting <sup>Screw or</sup> Worm Gears. shafts that are neither parallel nor meet, and the elements of whose teeth are helical or screw-like.

The circular pitch and number of teeth in a wheel being given, the diameter of the wheel and size of <sup>Sizing</sup> Blanks, &c. tooth parts are found as follows:

Dividing by 3.1416 is the same as multiplying by  $\frac{1}{3.1416}$ . Now  $\frac{1}{3.1416} = .3183$ ; hence, multiply the circumference of a circle by .3183 and the product will be the diameter of the circle. Multiply the circular pitch by .3183 and the product will be the same *part* of the

diameter of pitch circle that the circular pitch is of the circumference of pitch circle. This part or *modulus* is called a *diameter pitch*. There are as many diameter pitches contained in the diameter of pitch circle as there are teeth in the wheel.

A Diameter Pitch and the Addendum measure the same, radially. Most mechanics make the addendum of teeth equal to one diameter pitch. Hence we can designate this modulus or diameter pitch by the same letter as we do the addendum; that is, let  $s =$  a diameter pitch.

$.3183 P' = s$ , or circular pitch multiplied by  $.3183 = s$ , or a diameter pitch.

$Ns = D'$ , or number of teeth in a wheel, multiplied by a diameter pitch, equals diameter of pitch circle.

$(N + 2)s = D$ , or add 2 to the number of teeth, multiply the sum by  $s$  and product will be the whole diameter.

$\frac{t}{10} = f$ , or one tenth of thickness of tooth at pitch-line equals amount added to bottom of space for clearance.

Some mechanics prefer to make  $f$  equal to  $\frac{1}{16}$  of the working depth of teeth, or  $.0625 D'$ . One-tenth of the thickness of tooth at pitch-line is more than one-sixteenth of working depth, being  $.07854 D'$ .

**Example.**—Wheel 30 teeth,  $1\frac{1}{2}$ " circular pitch.  $P' = 1.5"$ ; then  $t = .75"$  or thickness of tooth equals  $\frac{3}{4}"$ .  $s = 1.5" \times .3183 = .4775" =$  a diameter pitch. (See tables of tooth parts, pages 64–67).

Sizes of Blank and Tooth Parts for Gear of 30 teeth,  $1\frac{1}{2}$  in. circular pitch.

$D' = 30 \times .4775" = 14.325" =$  diameter of pitch-circle.  
 $D = (30 + 2) \times .4775" = 15.280" =$  diameter of addendum circle.

$f = \frac{1}{10}$  of  $.75" = .075" =$  clearance at bottom of space.

$D'' = 2 \times .4775" = .9549" =$  working depth of teeth.

$D' + f = 2 \times .4775" + .075" = 1.0299" =$  whole depth of space.

$s + f = .4775" + .075" = .5525" =$  depth of space inside of pitch-line.

$D'' = 2s$  or the working depth of teeth is equal to two diameter pitches.

In making calculations it is well to retain the fourth place in the decimals, but when drawings are passed into the workshop, three places of decimals are sufficient.

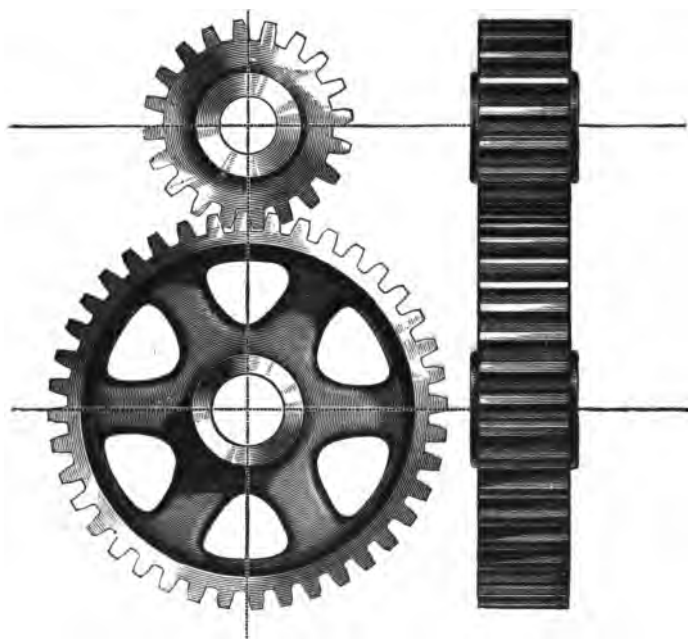


FIG. 5, SPUR GEARING.



Distance between centers of two Gears.

The distance between the centers of two wheels is evidently equal to the radius of pitch-circle of one wheel added to that of the other. The radius of pitch-circle is equal to  $s$  multiplied by one-half the number of teeth in the wheel.

Hence, if we know the number of teeth in two wheels, in mesh, and the circular pitch, to obtain the distance between centers we first find  $s$ ; then multiply  $s$  by one-half the sum of number of teeth in both wheels and the product will be distance between centers.

**Example.**—What is the distance between the centers of two wheels 35 and 60 teeth,  $1\frac{1}{4}$ " circular pitch. We first find  $s$  to be  $1\frac{1}{4}" \times .3183 = .3979"$ . Multiplying by 47.5 (one-half the sum of 35 and 60 teeth) we obtain 18.899" as the distance between centers.

Allowance for Shrinkage in Gear Castings.

**Pattern Gears** should be made large enough to allow for shrinkage in casting. In cast iron the shrinkage is about  $\frac{1}{8}$  inch in one foot. For gears one to two feet in diameter it is well enough to add simply  $\frac{1}{16}$  of diameter of finished gear to the pattern. In gears about six inches diameter or less, the moulder will generally rap the pattern in the sand enough to make any allowance for shrinkage unnecessary. In pattern gears the spaces between teeth should be cut wider than finished gear spaces to allow for rapping and to avoid having too much cleaning to do in order to have gears run freely. In cut patterns of iron it is generally enough to make spaces .015" to .02" wider. This makes clearance .03" to .04" in the patterns. Some moulders might want .06" to .07" clearance.

Metal Pattern Gears.

Metal patterns should be cut straight; they work better with no draft. It is well to leave about .005" to be finished from side of patterns after teeth are cut; this extra stock to be taken away from side where cutter comes through so as to take out places where stock is broken out. The finishing should be done with file or emery wheel, as turning in a lathe is likely to break out stock as badly as a cutter might do.

If cutters are kept sharp and care is taken when coming through the allowance for finishing is not necessary and the blanks may be finished before they are cut.

## CHAPTER III.

## SINGLE-CURVE GEARS OF 30 TEETH AND OVER.

Single-curve teeth are so called because they have <sup>Single Curve</sup> <sub>Teeth.</sub> but one curve by theory, this curve forming both face and flank of tooth sides. In any gear of thirty teeth and more, this curve can be a single arc of a circle whose radius is one-fourth the radius of the pitch circle. In gears of thirty teeth and more, a fillet is added at bottom of tooth, to make it stronger, equal in radius to one-sixth the widest part of tooth space.

A cutter formed to leave this fillet has the advantage of wearing longer than it would if brought up to a corner.

In gears less than thirty teeth this fillet is made the same as just given, and sides of teeth are formed with more than one arc, as will be shown in Chapter VI.

Having calculated the data of a gear of 30 teeth,  $\frac{3}{4}$  inch circular pitch (as we did in Chapter II. for  $1\frac{1}{2}'' = \frac{3}{4}''$ . <sup>Example of a</sup> <sub>Gear, N=30, P'</sub> pitch), we proceed as follows:

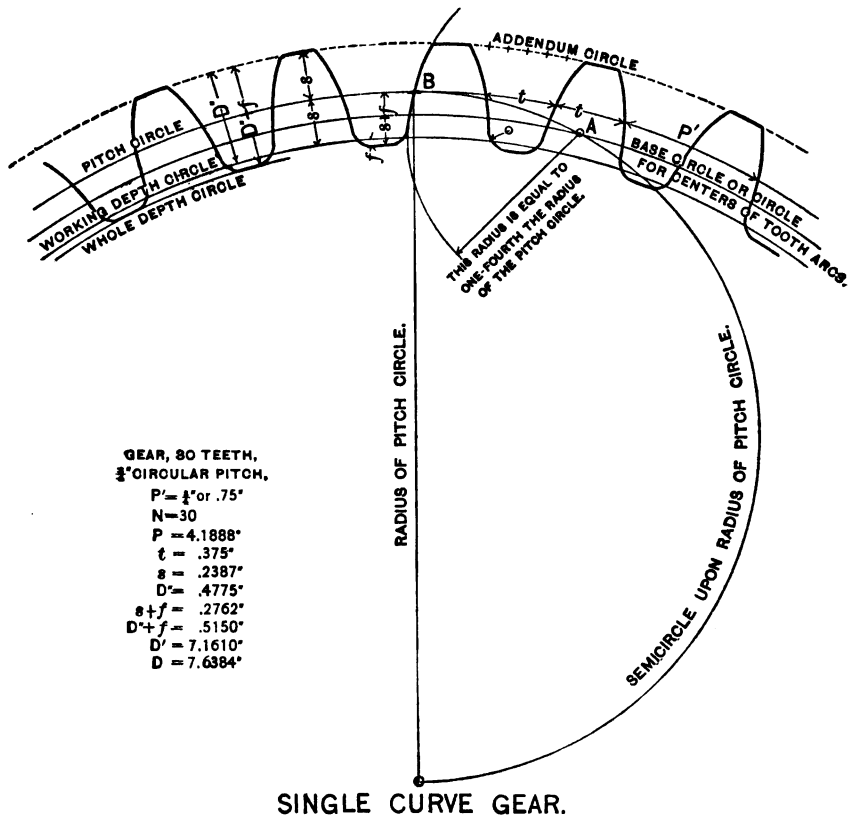
1. Draw pitch circle and point it off into parts equal to one-half the circular pitch. <sup>Geometrical</sup> <sub>Construction.</sub>   
 Fig. 6.

2. From one of these points, as at B, Fig. 6, draw radius to pitch circle, and upon this radius describe a semicircle; the diameter of this semicircle being equal to radius of pitch circle. Draw addendum, working depth and whole depth circles.

3. From the point B, Fig. 6, where semicircle, pitch circle and outer end of radius to pitch circle meet, lay off a distance upon semicircle equal to one-fourth the radius of pitch circle, shown in the figure at BA, and is laid off as a chord.

4. Through this new point at A, upon the semicircle, draw a circle concentric to pitch circle. This last is

Fig. 6



called the *base circle*, and is the one for centers of tooth arcs. In the system of single curve gears, we have adopted, the diameter of this circle is .968 of the diameter of pitch circle. Thus the base circle of any gear 1 inch pitch diameter by this system is .968". If the pitch circle is 2" the base circle will be 1.936."

5. With dividers set to one-quarter of the radius of pitch circle, draw arcs forming sides of teeth, placing one leg of the dividers in the base circle and letting the other leg describe an arc through a point in the pitch circle that was made in laying off the parts equal to one-half the circular pitch. Thus an arc is drawn about A as center through B.

6. With dividers set to one-sixth of the widest part of tooth space, draw the fillets for strengthening teeth at their roots. These fillet arcs should just touch the whole depth circle and the sides of teeth already described.

Single curve or involute gears are the *only* gears <sup>Peculiarity of Involute Gear-</sup> that can run at varying distance of axes and transmit <sup>ing.</sup> unvarying angular velocity. This peculiarity makes involute gears specially valuable for driving rolls or any rotating pieces, the distance of whose axes is likely to be changed.

The assertion that gears crowd harder on bearings <sup>Pressure on bearings.</sup> when of involute than when of other forms of teeth, has not been proved in actual practice.

Before taking next chapter, the learner should make several drawings of gears 30 teeth and more. Say <sup>Practice, before taking next chapter.</sup> make 35 and 70 teeth  $1\frac{1}{2}"$  P'. Then make 40 and 65 teeth  $\frac{7}{8}"$  P'.

An excellent practice will be to make drawing on cardboard or Bristol-board and cut teeth to lines, thus making paper gears; or, what is still better, make them of sheet metal. By placing these in mesh the learner can test the accuracy of his work.

## CHAPTER IV.

RACK TO MESH WITH SINGLE-CURVE GEARS HAVING  
30 TEETH AND OVER.

Diagram,  
made prepara-  
tory to drawing  
a Rack.

This gear (Fig. 7) is made precisely the same as gear in Chapter III. It makes no difference in which direction the construction radius is drawn, so far as obtaining form of teeth and making gear are concerned.

Here the radius is drawn perpendicular to pitch line of rack and through one of the tooth sides, B. A semi-circle is drawn on each side of the radius of the pitch circle.

The points A and A' are each distant from the point B, equal to one-fourth the radius of pitch circle and correspond to the point A in Fig. 6.

In Fig. 7 add two lines, one passing through B and A and one through B and A'. These two lines form angles of  $75\frac{1}{2}^\circ$  (degrees) with radius BO. Lines BA and BA' are called lines of pressure. The sides of rack teeth are made perpendicular to these lines.

Rack.

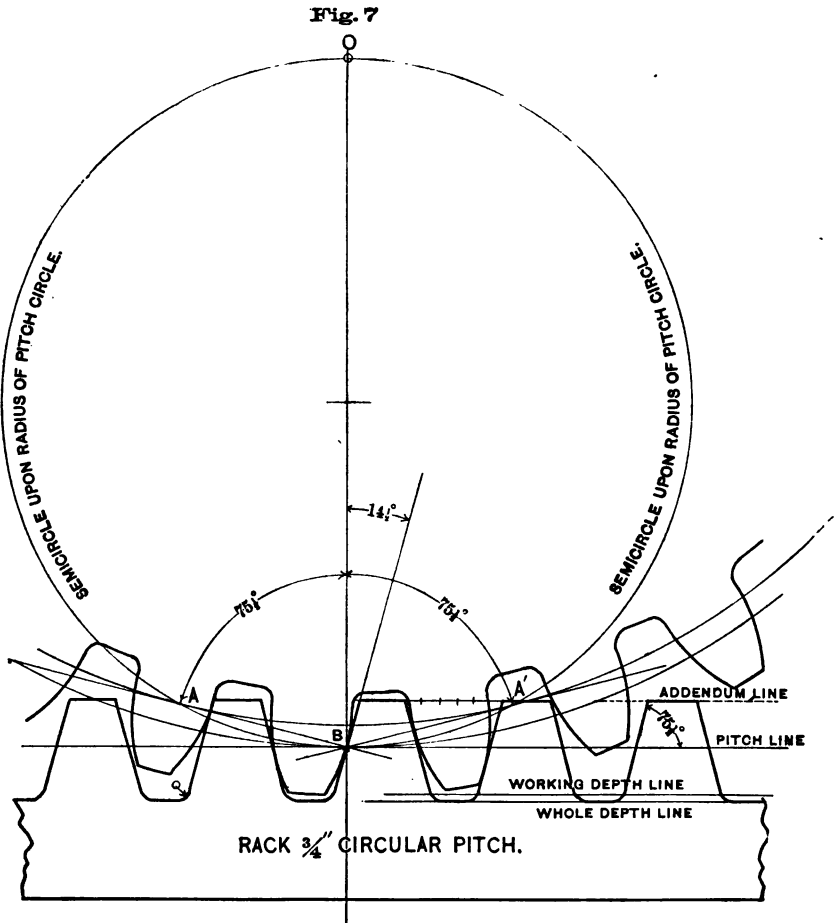
Construction  
of Pitch Line of  
Rack.

A Rack is a straight piece, having teeth to mesh with a gear. A rack may be considered as a gear of infinitely long radius. The circumference of a circle approaches a straight line as the radius increases, and when the radius is infinitely long any finite part of the circumference is a straight line. The pitch line of a rack, then, is merely a straight line just touching the pitch circle of a gear meshing with the rack. The thickness of teeth, addendum and depth of teeth below pitch line are calculated the same as for a wheel. (For pitches in common use, see table of tooth parts.)

A rack to mesh with a single-curve gear of 30 teeth or more is drawn as follows:

Rack.  
Fig. 7.

1. Draw straight pitch line of rack; also draw addendum line, working depth line and whole depth line, each parallel to the pitch line (see Fig. 7).



RACK TO MESH WITH SINGLE CURVE GEAR  
HAVING 30 TEETH AND OVER.

2. Point off the pitch line into parts equal to one-half the circular pitch, or  $=t$ .

3. Through these points draw lines at an angle of  $75\frac{1}{2}^\circ$  with pitch lines, alternate lines slanting in opposite directions. The left-hand side of each rack tooth is perpendicular to the line BA. The right-hand side of each rack tooth is perpendicular to the line BA'.

4. Add fillets at bottom of teeth equal to  $\frac{1}{4}$  of the width of spaces between the rack teeth at the addendum line.

Angle for  
sides of Rack  
Teeth.

The sketch, Fig. 8, will show how to obtain angle of sides of rack teeth, directly from pitch line of rack, without drawing a gear in mesh with the rack.

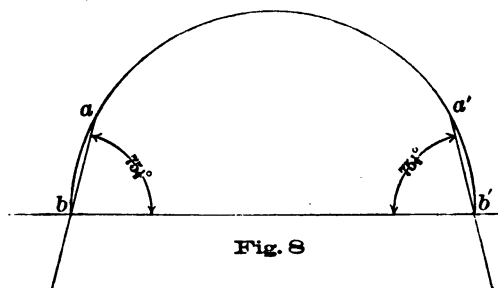


Fig. 8

Upon the pitch line  $b b'$ , draw any semicircle— $b a a' b'$ . From point  $b$  lay off upon the semicircle the distance  $b a$ , equal to one-quarter of the diameter of semicircle, and draw a straight line through  $b$  and  $a$ .

This line,  $b a$ , makes an angle of  $75\frac{1}{2}^\circ$  with pitch line  $b b'$ , and can be one side of rack tooth. The same construction,  $b' a'$ , will give the inclination  $75\frac{1}{2}^\circ$  in the opposite direction for the other side of tooth.

The sketch, Fig. 9, gives the angle of sides of a tool for planing out spaces between rack teeth. Upon any line OB draw circle OABA'. From B lay off distance BA and BA', each equal to one-quarter of diameter of the circle.

Draw lines OA and OA'. These two lines form an angle of  $29^\circ$ , and are right for inclination of sides of rack tool.

Make end of rack tool .31 of circular pitch, and then round the corners of the tool to leave fillets at the bottom of rack teeth. Width of Rack  
Tool at end.

Thus, if the circular pitch of a rack is  $1\frac{1}{2}$ " and we multiply by .31, the product .465" will be the width of tool at end for rack of this pitch before corners are taken off. This width is shown at  $xy$ .

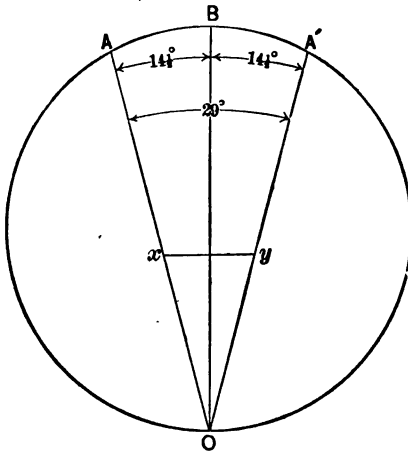


Fig. 9

This sketch and the foregoing rule are also right for Worm Thread  
Tool. a worm-thread tool, but a worm-thread tool is not usually rounded for fillet. In cutting worms, leave width of top of thread .335 of the circular pitch. When this is done, the depth of thread will be right.



## CHAPTER V.

DIAMETRAL PITCH—SIZING BLANKS AND TEETH—DISTANCE  
BETWEEN CENTERS OF WHEELS.

In making drawings of gears, and in cutting racks, When it is necessary to know the circular pitch, both on account of spacing teeth and calculating their strength.

It would be more convenient to express the circular pitch in whole inches, and the most natural divisions of an inch, as 1" P',  $\frac{3}{4}$ " P',  $\frac{1}{2}$ " P', and so on. But as the circumference of the pitch circle must contain the circular pitch some whole number of times, corresponding to the number of teeth in the gear, the *diameter* of the pitch circle will often be of a size not readily measured with a common rule. This is because the circumference of a circle is equal to 3.1416 times the diameter, or the diameter is equal to the circumference multiplied by .3183.

In practice, it is better that the diameter should be of some size conveniently measured. The same applies to the distance between centers. Hence it is generally more convenient to assume the pitch in terms of the diameter. In Chapter II. was given a definition of a diameter pitch, and also how to get a diameter pitch from the circular pitch.

We can also assume a diameter pitch and pass to its equivalent circular pitch. If the circumference of the pitch circle is divided by the number of teeth in the gear, the quotient will be the circular pitch. In the same manner, if the *diameter* of the pitch circle is divided by the number of teeth, the quotient will be a diameter pitch. Thus, if a gear is 12 inches pitch diameter and has 48 teeth, dividing 12" by 48, the quotient  $\frac{1}{4}$ " is a diameter pitch of this gear. In prac-

tice, a diameter pitch is taken in some convenient part of an inch, as  $\frac{1}{2}$ " diameter pitch, and so on. It is convenient in calculation to designate one of these diameter pitches by  $s$ , as in Chapter II. Thus, for  $\frac{1}{2}$ " diameter pitch,  $s$  is equal to  $\frac{1}{2}$ ". Generally, in speaking of diameter pitch, the denominator of the fraction only is named.  $\frac{1}{3}$ " diameter pitch is then called 3 *diametral pitch*. That is, it has been found more convenient to take the *reciprocal* of a diameter pitch in making calculation. The reciprocal of a number is 1, divided by that number. Thus the reciprocal of  $\frac{1}{4}$  is 4, because  $\frac{1}{4}$  goes into 1 four times.

Abbreviation  
of Diameter  
Pitch.

Reciprocal of  
a Number.

Hence, we come to the common definition :

**DIAMETRAL PITCH** is the number of teeth to *one inch* of diameter of pitch circle. Let this be denoted by  $P$ . Thus,  $\frac{1}{4}$ " diameter pitch we would call 4 *diametral pitch* or 4  $P$ , because there would be 4 teeth to every inch in the diameter of pitch circle. The circular pitch and the different parts of the teeth are derived from the diametral pitch as follows.

Diametral  
Pitch.

$\frac{3.1416}{P} = P'$ , or 3.1416 divided by the diametral pitch is equal to the circular pitch. Thus to obtain the circular for 4 diametral pitch, we divide 3.1416 by 4 and get .7854 for the circular pitch, corresponding to 4 diametral pitch.

To obtain Cir-  
cular Pitch  
from Diami-  
tral Pitch.

In this case we would write  $P=4, P'=.7854", s=\frac{1}{4}"$ .  $\frac{1"}{P}=s$ , or one inch divided by the number of teeth to an inch, gives distance on diameter of pitch circle occupied by *one* tooth. The addendum or face of tooth is the same distance as  $s$ .

$\frac{1}{s}=P$ , or one inch divided by the distance occupied by one tooth equals number of teeth to one inch.

$\frac{1.57}{P}=t$ , or 1.57 divided by the diametral pitch gives thickness of tooth at pitch line. Thus, thickness of teeth along the pitch line for 4 diametral pitch is .392".

Calculations  
for Sizes of  
Blanks and  
tooth parts  
when the Dia-  
metral Pitch is  
given.

$\frac{N}{P}=D'$ , or number of teeth in a gear divided by the diametral pitch equals diameter of the pitch circle. Thus for a wheel, 60 teeth, 12  $P$ , the diameter of pitch circle will be 5 inches.

$\frac{N+2}{P}=D$ , or add 2 to the number of teeth in a wheel and divide the sum by the diametral pitch, and the

quotient will be the *whole diameter* of the gear or the diameter of the addendum circle. Thus, for 60 teeth, 12 P, the diameter of gear blank will be  $5\frac{1}{2}$  inches.

$\frac{N}{D} = P$ , or number of teeth divided by diameter of pitch circle in inches, gives the diametral pitch or number of teeth to one inch. Thus, in a wheel, 24 teeth, 3 inches pitch diameter, the diametral pitch is 8.

$\frac{N+2}{D} = P$ , or add 2 to the number of teeth; divide the sum by the whole diameter of gear, and the quotient will be the diametral pitch. Thus, for a wheel  $3\frac{1}{8}$ " diameter, 14 teeth, the diametral pitch is 5.

$P D' = N$ , or diameter of pitch circle, multiplied by diametral pitch equals number of teeth in the gear. Thus, in a gear, 5 pitch, 8" pitch diameter, the number of teeth is 40.

A Diametral Pitch.

In future, when we speak of a *diameter pitch*, we shall mean the addendum distance or  $s$ . If we speak of so many diameter pitches, we shall mean so many

The Diametral Pitch.

times  $s$ , ( $\frac{1''}{P} = s$ ). When we say *the diametral pitch* we shall mean the number of teeth to one inch of diameter of pitch circle, or  $P$ , ( $\frac{1''}{s} = P$ ).

To obtain Diametral Pitch from Circular Pitch.

When the circular pitch is given, to find the corresponding diametral pitch, divide 3.1416 by the circular pitch. Thus 1.57 P is the diametral pitch corresponding to 2-inch circular pitch, ( $\frac{3.1416}{2} = P$ ).

Example.

What diametral pitch corresponds to  $\frac{1}{2}$ " circular pitch? Remembering that to divide by a fraction we multiply by the denominator and divide by the numerator, we obtain 6.28 as the quotient of 3.1416 divided by  $\frac{1}{2}$ . 6.28 P, then, is the diametral pitch corresponding to  $\frac{1}{2}$  circular pitch. This means that in a gear of  $\frac{1}{2}$  inch circular pitch there are six and twenty-eight one hundredths teeth to every inch in the diameter of the pitch circle. In the table of tooth parts the diametral pitches corresponding to circular pitches are carried out to four places of decimals, but in practice two places of decimals are enough.

When two gears are in mesh, so that their pitch circles just touch, the distance between their axes or centers is equal to the sum of the radii of the two

gears. The number of the *diameter* pitches between centers is equal to half the sum of number of teeth in both gears. This principle is the same as given in Chapter II., page 6, but when the diametral pitch and numbers of teeth in two gears are given, *add together the numbers of teeth in the two wheels and divide half the sum by the diametral pitch. The quotient is the center distance.* Rule to find Distance between Centers.

A gear of 20 teeth, 4 P, meshes with a gear of 50 Example.  
teeth: what is the distance between their axes or centers? Adding 50 to 20 and dividing half the sum by 4, we obtain  $8\frac{3}{4}$ " as the center distance.

The term *diametral pitch* is also applied to a rack. Thus, a rack 3 P, means a rack that will mesh with a gear of 3 diametral pitch.

It will be seen that if the expression for a diameter pitch has any number except 1 for a numerator, we cannot express the diametral pitch by naming the denominator only. Thus, if the addendum or a diameter pitch is  $\frac{4}{10}$ ", the diametral pitch will be  $2\frac{1}{2}$ , because 1 divided by  $\frac{4}{10}$  equals  $2\frac{1}{2}$ . Fractional Diametral Pitch.

## CHAPTER VI.

## SINGLE-CURVE GEARS HAVING LESS THAN 30 TEETH—GEARS AND RACKS TO MESH WITH GEARS HAVING LESS THAN 30 TEETH.

Construction,  
Fig. 10.

In Fig. 10, the construction of the rack is the same as the construction of the rack in Chapter IV. The gear in Fig. 10 is drawn from base circle out to addendum circle, by the same method as the gear in Chapter III., but the spaces inside of base circle are drawn as follows:

Flanks of  
Gears in low  
numbers of  
Teeth.

In gears, 12 and 13 teeth, the sides of spaces inside of base circle are parallel for a distance not more than  $\frac{1}{8}$  of a diameter pitch, or  $\frac{1}{8}s$ ; gears 14, 15 and 16 teeth not more than  $\frac{1}{4}s$ ; 17 to 20 teeth, not more than  $\frac{1}{2}s$ . In gears with more than 20 teeth the parallel construction is omitted.

Construction  
of Fig. 10 con-  
tinued.

Then, with one leg of dividers in pitch circle in center of next tooth,  $e$ , and other leg just touching one of the parallel lines at  $b$ , continue the tooth side into  $c$ , until it will touch a fillet arc, whose radius is  $\frac{1}{2}$  the width of space at the addendum circle. The part,  $b'c'$ , is an arc from center of tooth  $g$ , etc. The flanks of teeth or spaces in gear, Fig. 11, are made the same as those in Fig. 10.

Internal Gear.

This rule is merely conventional or not founded upon any principle other than the judgment of the designer, to effect the object to have spaces as wide as practicable, just below or inside of base circle, and then strengthen flank with as large a fillet as will clear addenda of any gear. If flanks in any gear will clear addenda of a rack, they will clear addenda of any other gear, except internal gears. An internal gear is one having teeth upon the inner side of a rim or ring. Now, it will be seen that the gear, Fig. 10, has teeth

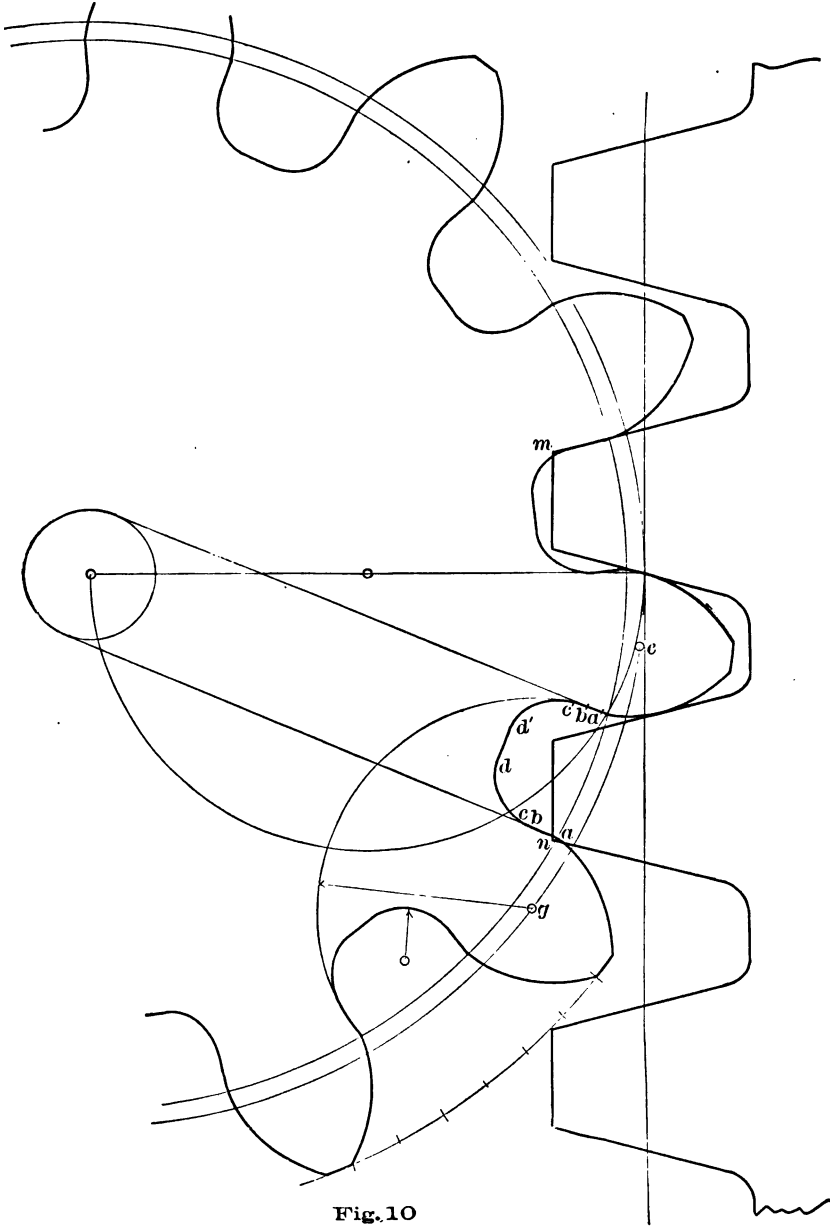


Fig. 10

Rounding of  
Addenda of  
Teeth.

too much rounded at the points or at the addendum circle. In gears of pitch coarser than 10 to inch (10 P), and having less than 30 teeth, this rounding becomes objectionable. This rounding occurs, because in these gears arcs of circles depart too far from the true involute curve, being so much that points of teeth get no bearing on flanks of teeth in other wheels.

Approximation to True Involute.

In gear, Fig. 11, the teeth outside of base circle are made as nearly true involute as a workman will be able to get without special machinery. This is accomplished as follows: draw three or four tangents to the base circle,  $i i', j j', k k', l l'$ , letting the points of tangency on base circle  $i', j', k', l'$  be about  $\frac{1}{4}$  or  $\frac{1}{2}$  the circular pitch apart; the first point,  $i'$ , being distant from  $i$ , equal to  $\frac{1}{4}$  the radius of pitch circle. With dividers set to  $\frac{1}{4}$  the radius of pitch circle, placing one leg in  $i'$ , draw the arc,  $a' i j$ ; with one leg in  $j'$ , and radius  $j' j$ , draw  $j k$ ; with one leg in  $k'$ , and radius  $k' k$  draw  $k l$ . Should the addendum circle be outside of  $l$ , the tooth side can be completed with the last radius,  $l' l$ . The arcs,  $a' i j$ ,  $j k$  and  $k l$ , together form a very close approximation to a true involute from the base circle,  $i' j' k' l'$ . The *exact* involute for gear teeth is the curve made by the end of a band when unwound from a cylinder of the same diameter as base circle.

The foregoing operation of drawing tooth sides, although tedious in description, is very easy of practical application.

Rounding of  
Addenda of  
Rack.

It will also be seen that the addenda of rack teeth in Fig. 10, interfere with the gear-teeth flanks, as at  $m n$ ; to avoid this interference, the teeth of rack, Fig. 11, are rounded at points or addenda.

Templets  
necessary for  
Rounding off  
Points of teeth.

It is also necessary to round off the points of involute teeth in high-numbered gears, when they are to interchange with low-numbered gears. In interchangeable sets of gears the lowest-numbered pinion is usually 12. Just how much to round off is best learnt by making templets of a few teeth out of thin metal or cardboard, for the gear and rack, or, two gears required, and fitting addenda of teeth to clear flanks. However accurate we may make a diagram, it is quite

SINGLE CURVE GEAR, 2 P., 12 TEETH.  
IN MESH WITH RACK.

$P = 2$   
 $N = 12$   
 $P' = 1.57''$   
 $t = .7854''$   
 $s = .500''$   
 $D' = 1.000''$   
 $s + f = .5785''$   
 $D' + f = 1.078''$   
 $D' = 6''$   
 $D = 7''$

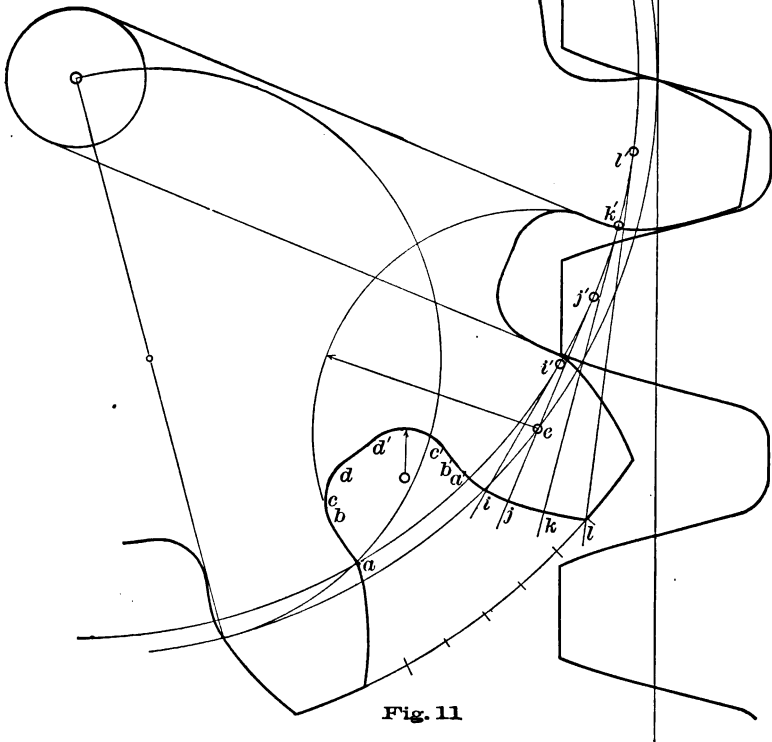


Fig. 11



as well to make templets in order to shape cutters accurately.

A set of cutters for interchangeable gears will give good results in finer pitches, made to Fig. 6, in Chapter III., for 30 teeth and over, and to Fig. 10, for less than 30 teeth, arranged upon the following:

Diagrams for  
a Set of Cut-  
ters.

For cutting 12 and 13 teeth, make cutter to diagram of 14 teeth.

For 14 to 16 teeth, diagram of 16 teeth.

" 17 "	20 "	"	"	" 18 "
" 21 "	25 "	"	"	" 22 "
" 26 "	34 "	"	"	" 28 "
" 35 "	54 "	"	"	" 38 "
" 55 "	134 "	"	"	" 55 "
" 135 "	rack, "	"	"	" 100 "

Cut a rack with cutter made to diagram for 100 teeth.

For gears, 10 P and coarser, it is better to make cutters to corrected diagrams, as in Fig. 11. When corrected diagrams are made, as in Fig. 11, take the following:

For 12 and 13 teeth, diagram of 12 teeth.

" 14 to 16 "	"	"	" 14 "
" 17 "	20 "	"	" 17 "
" 21 "	25 "	"	" 21 "
" 26 "	34 "	"	" 26 "
" 35 "	54 "	"	" 35 "
" 55 "	134 "	"	" 55 "
" 135 "	rack, "	"	" 135 "

Templets for large gears must be fitted to run with 12 teeth, etc.

We obtain the form for involute cutters by special machinery.

## CHAPTER VII.

## DOUBLE-CURVE TEETH—GEAR, 15 TEETH—RACK.

In double-curve teeth the formation of tooth sides changes at the pitch line. In all gears the part of teeth outside of pitch line is convex; in some gears the sides of teeth inside pitch line are convex; in some, radial; in others, concave. Convex faces and concave flanks are most familiar to mechanics. In interchangeable sets of gears, *one* gear in each set, or of each pitch, has radial flanks. In the best practice, *this gear has fifteen teeth*. Gears with more than fifteen teeth, have concave flanks; gears with less than fifteen teeth, have convex flanks. Fifteen teeth is called the *Base* of this system.

We will first draw a gear of fifteen teeth. This fifteen-tooth construction enters into gears of any number of teeth and also into racks. Let the gear be 3 P. Having obtained data, we proceed as follows:

1. Draw pitch circle and point it off into parts equal to one-thirtieth of the circumference, or equal to thickness of tooth =  $t$ .

2. From the center, through one of these points, as at T, Fig. 12, draw line O T A. Draw addendum and whole-depth circles.

3. About this point, T, with same radius as 15-tooth pitch circle, describe arcs A K and O  $k$ . For any other double-curve gear of 3 P., the radius of arcs, A K and O  $k$ , will be the same as in this 15-tooth gear =  $2\frac{1}{2}t$ . In a 15-tooth gear, the arc, O  $k$ , passes through the center O, but for a gear having any other number of teeth, this construction arc does not pass through center of gear. Of course, the 15-tooth radius of arcs, A K and O  $k$ , is always taken from the pitch we are working with.

All Double-curve Tooth Faces are Convex.

Construction of Fig. 12.

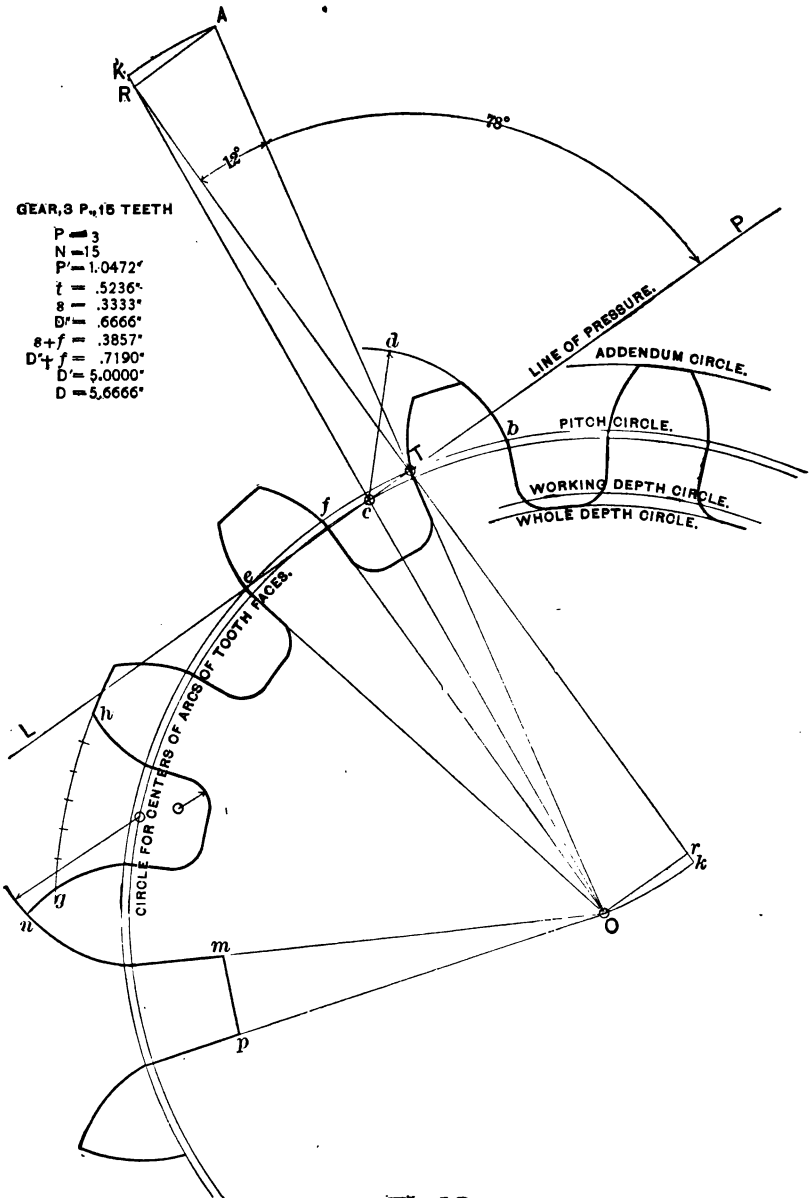


Fig. 12  
DOUBLE CURVE GEAR.

4. Upon these arcs on opposite sides of lines  $O T A$ , lay off tooth thickness,  $A K$  and  $O k$ , and draw line  $K T k$ .

5. Perpendicular to  $K T k$ , draw line of pressure,  $L T P$ ; also through  $O$  and  $A$ , draw lines  $A R$  and  $O r$ , perpendicular to  $K T k$ . The line of pressure is at an angle of  $78^\circ$  with the radius of gear.

6. From  $O$ , draw a line  $O R$  to intersection of  $A R$  with  $K T k$ . Through point  $c$ , where  $O R$  intersects  $L P$ , describe a circle about the center,  $O$ . *In this circle one leg of dividers is placed to describe tooth faces*

7. The radius,  $c d$ , of arc of tooth faces is the straight distance from  $c$  to tooth-thickness point,  $b$ , on the other side of radius,  $O T$ . With this radius,  $c b$ , describe both sides of tooth faces.

8. Draw flanks of all teeth radial, as  $O e$  and  $O f$ . The *base gear*, 15 teeth only, has radial flanks.

9. With radius equal to one sixth of the widest part of space, as  $g h$ , draw fillets at bottom of teeth.

The foregoing is a close approximation to epicycloidal teeth. To get exact teeth, make two 15 tooth gears of thin metal. Make addenda long enough to come to a point, as at  $n$  and  $q$ . Make radial flanks, as at  $m$  and  $p$ , deep enough to clear addenda when gears are in mesh. First finish the flanks, then fit the long addenda to the flanks when gears are in mesh.

When these two templet gears are alike, the centers are the right distance apart and the teeth interlock without backlash, they are exact. One of these templet gears can now be used to test any other templet gear of the same pitch.

Gears and racks will be right when they run correctly with one of these 15-tooth templet gears. Five or six teeth are enough to make in a gear templet.

**DOUBLE-CURVE RACK.**—Let us draw a rack 3 P. Having obtained data of teeth we proceed as follows:

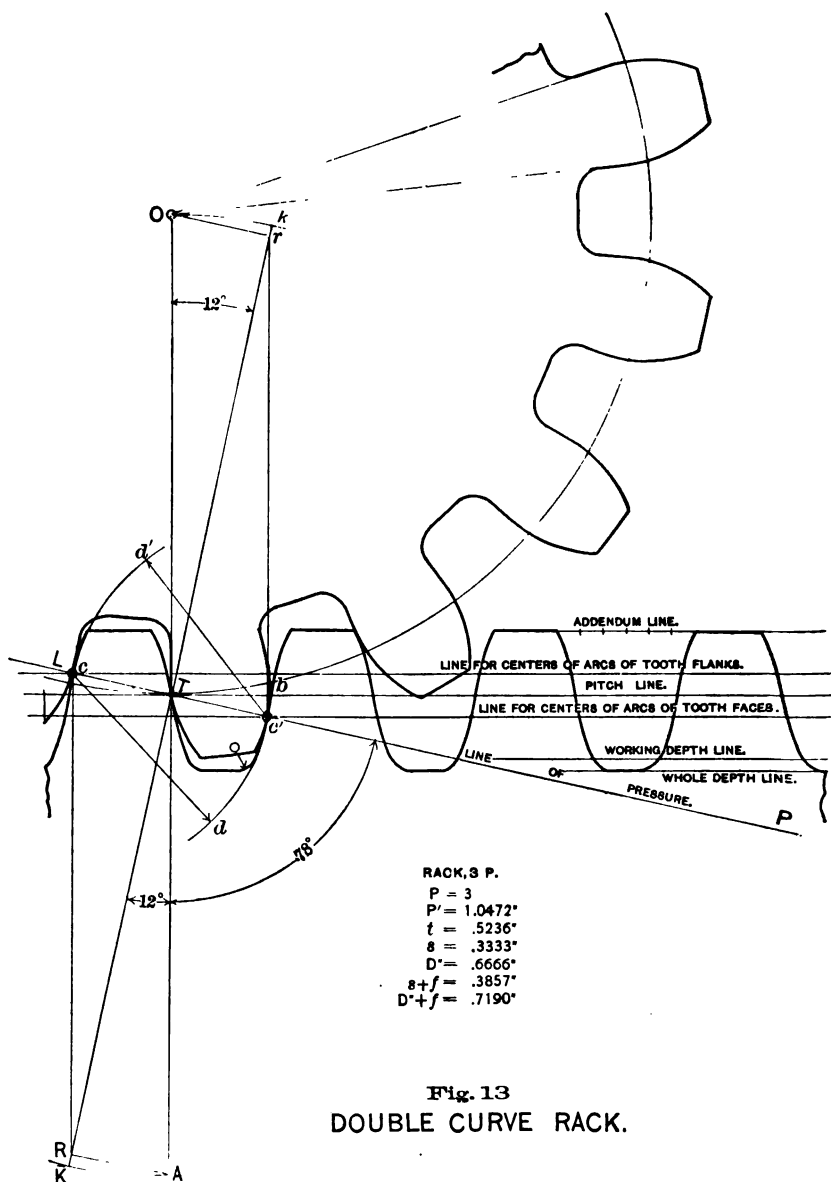
1. Draw pitch line and point it off in parts equal to one-half the circular pitch. Draw addendum and whole-depth lines.

2. Through one of the points, as at  $T$ , Fig. 13, draw line  $O T A$  perpendicular to pitch line of rack.

Approximation to Epicycloidal Teeth.

Standard Templets.

Double-curve Rack, Fig. 13.



3. About T make precisely the same construction as was made about T in Fig. 12. That is, with radius of 15-tooth pitch circle and center T draw arcs  $O k$  and  $A K$ ; make  $O k$  and  $A K$  equal to tooth thickness; draw  $K T k$ ; draw  $O r$ ,  $A R$ , and line of pressure, each perpendicular to  $K T k$ .

4. Through R and  $r$ , draw lines parallel to  $O A$ . Through intersections  $c$  and  $c'$  of these lines, with pressure line  $L P$ , draw lines parallel to pitch line.

5. In these last lines place leg of dividers, and draw faces and flanks of teeth as in sketch.

6. The radius  $c' d'$  of rack-tooth faces is the same length as radius  $c d$  of rack-tooth flanks, and is the straight distance from  $c$  to tooth-thickness point  $b$  on opposite side of line  $O A$ .

7. The radius for fillet at bottom of rack teeth is equal to  $\frac{1}{4}$  of the widest part of tooth space. This radius can be varied to suit the judgment of the designer, so long as a fillet does not interfere with teeth of engaging gear.

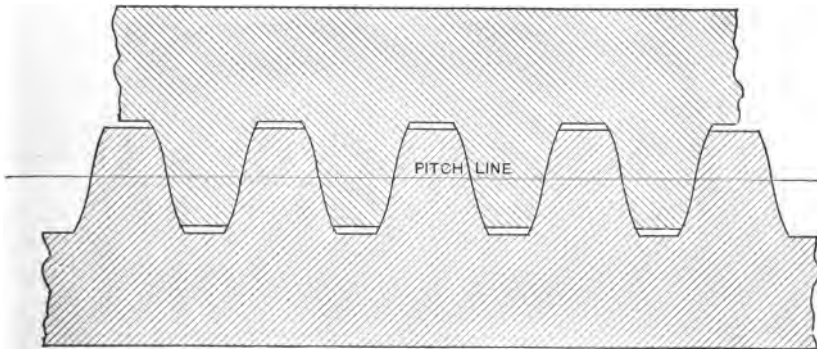


Fig. 14

Racks of the same pitch, to mesh with interchangeable gears, should be alike when placed side by side, and fit each other when placed together as in Fig. 14.

In Fig. 13, a few teeth of a 15-tooth wheel are shown in mesh with the rack.

## CHAPTER VIII.

DOUBLE-CURVE GEARS, HAVING MORE AND LESS THAN  
15 TEETH—ANNULAR GEARS.

Construction  
of Fig. 15.

Let us draw two gears, 12 and 24 teeth, 4 P, in mesh. In Fig. 15 the construction lines of the lower or 24-tooth gear are full. The upper or 12-tooth gear construction lines are dotted. The line of pressure, L P, and the line K T  $k$  answer for both gears. The arcs A K and O  $k$  are described about T. The radius of these arcs is the radius of pitch circle of a gear 15 teeth 4 pitch. The length of arcs A K and O  $k$  is the tooth thickness for 4 P. The line K T  $k$  is obtained the same as in Chapter VII. for all double-curve gears, the distances only varying as the pitch. Having drawn the pitch circles, the line K T  $k$ , and, perpendicular to K T  $k$ , the lines A R, O  $r$  and the line of pressure L T P, we proceed with the 24-tooth gear as follows:

1. From center C, through  $r$ , draw line intersecting line of pressure in  $m$ . Also draw line from center C to R, crossing the line of pressure L P at  $c$ .

2. Through  $m$  describe circle concentric with pitch circle about C. This is the circle in which to place one leg of dividers to describe flanks of teeth.

3. The radius,  $m n$ , of flanks is the straight distance from  $m$  to the first tooth-thickness point on other side of line of centers, C C', at  $v$ . The arc is continued to  $n$ , to show how constructed. This method of obtaining radius of double-curve tooth flanks applies to all gears having more than fifteen teeth.

4. The construction of tooth faces is similar to 15-tooth wheel in Chapter VII. That is: Draw a circle through  $c$  concentric to pitch circle; in this circle place one leg of dividers to draw tooth faces, the radius of tooth faces being  $c b$ .

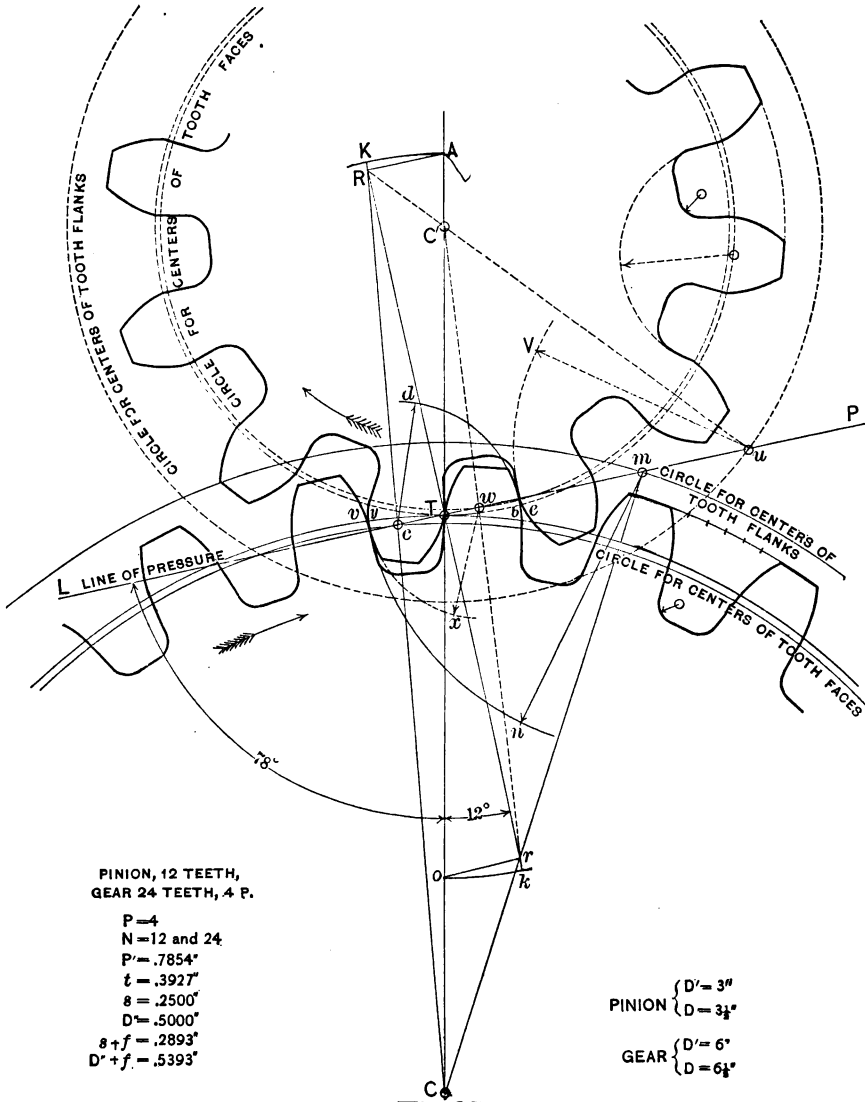


Fig. 15.  
DOUBLE CURVE GEARS IN MESH.



Construction  
of Fig. 15 con-  
tinued.

5. The radius of fillets at roots of teeth is equal to one-sixth the width of space at addendum circle.

Flanks for 12,  
13 and 14 Teeth.

**The constructions for flanks of 12, 13 and 14 teeth are similar to each other and as follows:**

1. Through center,  $C'$ , draw line from  $R$ , intersecting line of pressure in  $u$ . Through  $u$  draw circle about  $C'$ . In this circle one leg of dividers is placed for drawing flanks.

2. The radius of flanks is the distance from  $u$  to the first tooth-thickness point,  $e$ , on the *same side* of  $C T C'$ . This gives convex flanks. The arc is continued to  $V$ , to show construction.

3. This arc for flanks is continued in or toward the center, only about one-sixth of the working depth (or  $\frac{1}{8} s.$ ); the lower part of flank is similar to flanks of gear in Chapter VI.

4. The faces are similar to those in 15-tooth gear, Chapter VII., and to the 24-tooth gear in the foregoing, the radius being  $w y$ ; the arc is continued to  $x$ , to show construction.

Annular Gears.

**ANNULAR GEARS.** Gears with teeth inside of a rim or ring are called Annular or Internal Gears. The construction of tooth outlines is similar to the foregoing, but the *spaces* of a spur external gear become the *teeth* of an annular gear.

Prof. MacCord has shown that in the system just described, the pinion meshing with an annular gear, must differ from it by *at least* fifteen teeth. Thus, a gear of 24 teeth cannot work with an annular gear of 36 teeth, but it will work with annular gears of 39 teeth and more. An annular gear differing from its mate by less than 15 teeth can be made. This will be shown in Part II.

Annular-gear patterns require more clearance for moulding than external or spur gears.

Pinions.

In speaking of different-sized gears, the smallest ones are often called "pinions."

The angle of pressure in all gears except involute, constantly changes.  $78^\circ$  is the pressure angle in double-curve, or epicycloidal gears for an instant only; in our example, it is  $78^\circ$  when one side of a

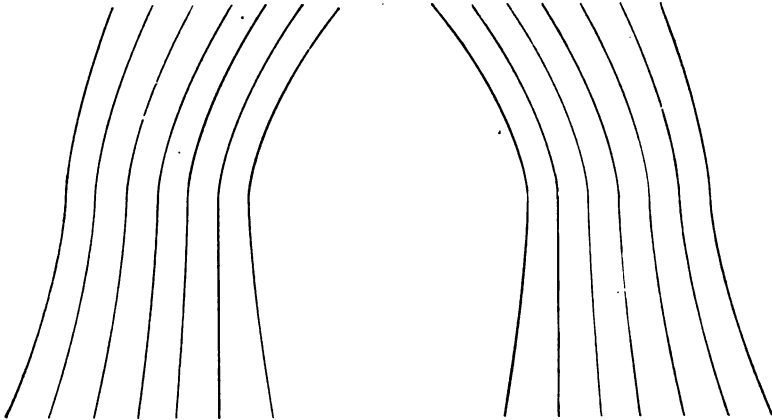
tooth reaches the line of centers, and the pressure against teeth is applied in the direction of the arrows.

The pressure angle of involute gears does not change. An explanation of the term angle of pressure is given in Part II.

We obtain the forms for epicycloidal gear cutters by means of a machine called the Odontom Engine. This machine will cut original gears with theoretical accuracy.

It has been thought best to make 24 gear cutters <sup>24 Double-curve Gear Cutters for each Pitch.</sup> for each pitch. This enables us to fill any requirement of gear-cutting very closely, as the range covered by any one cutter is so small that it is exceedingly near to the exact shape of all gears so covered.

Of course, a cutter can be *exactly* right for only one gear. Special cutters can be made, if desired.



# 1 PITCH TOOTH CURVES

from the  
ODONTOM ENGINE.

## CHAPTER IX.

### BEVEL-GEAR BLANKS.

---

The pitch of bevel gears is always figured at the largest pitch diameter.

Most bevel gears connect shafts that are at right angles with each other, and unless stated otherwise, we always understand they are so wanted.

Description of  
Diagram, Fig.  
16, showing Con-  
struction of  
Bevel-gear  
Blanks.

The directions given in the following, apply to any angle (in our sketch, Fig. 16, the axes are at right angles). Having decided upon the pitch, numbers of teeth and angle of shafts:

1. Draw center lines of shafts,  $A O B$  and  $C O D$ , at angle required.

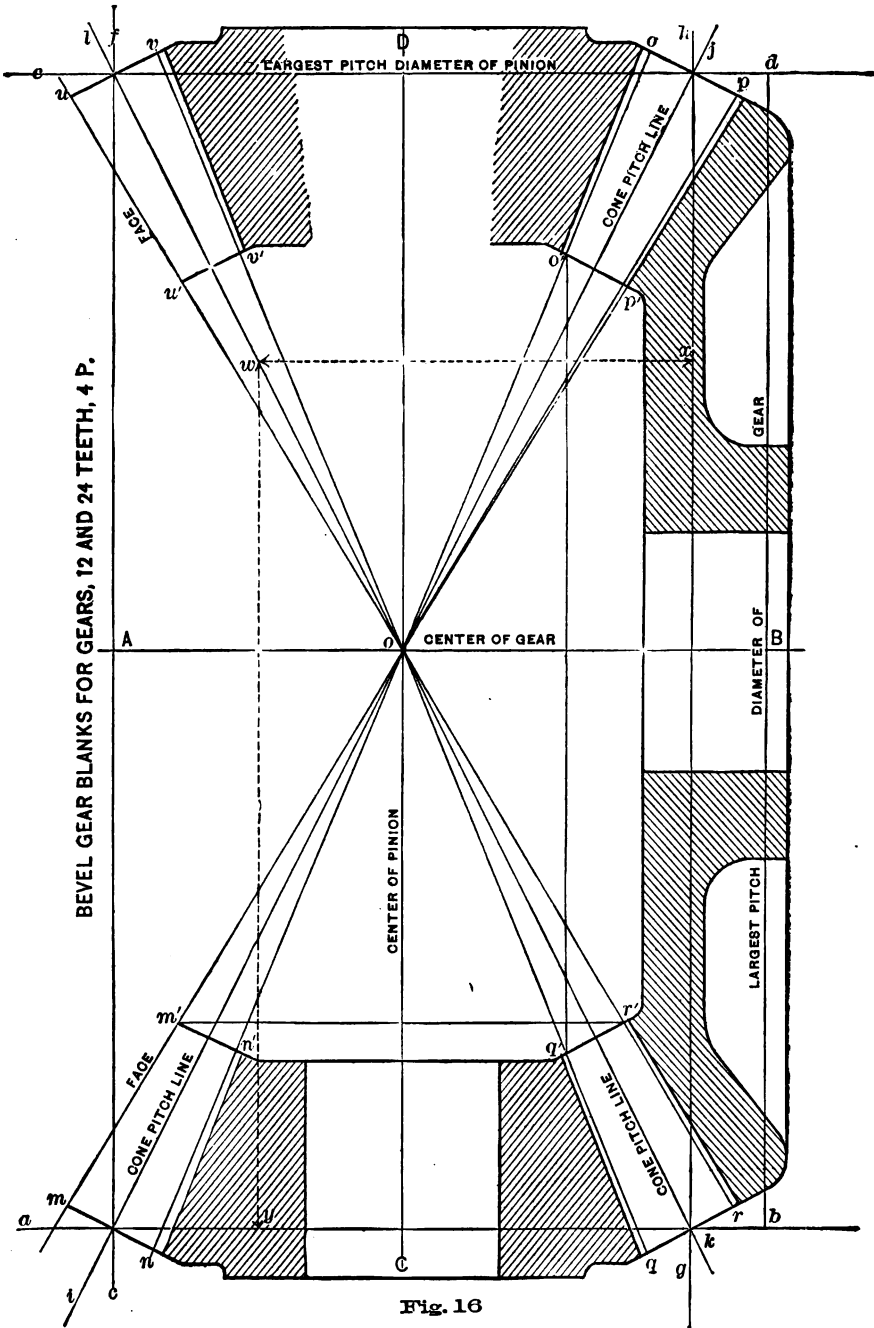
2. Parallel to  $A O B$ , draw lines  $a b$  and  $c d$ , each distant from  $A O B$ , equal to half the largest pitch diameter of one gear. For 24 teeth, 4 pitch, this half largest pitch diameter is 3".

3. Parallel to  $C O D$ , draw lines  $e f$  and  $g h$ , distant from  $C O D$ , equal to half the largest pitch diameter of other gear. For gear, 12 teeth, 4 pitch, this half largest pitch diameter is  $1\frac{1}{2}$ ".

4. At intersections of these four lines, draw lines  $O i$ ,  $O j$ ,  $O k$ , and  $O l$ ; these lines give the size and shape of pitch cones. We call them "Cone Pitch Lines."

5. Perpendicular to cone-pitch lines and through intersection of lines  $a b$ ,  $c d$ ,  $e f$ ,  $g h$ , draw lines  $m n$ ,  $o p$ ,  $q r$ . We have also drawn  $u v$  to show that another gear can be drawn from same diagram. When shafts are at right angles, four gears, two of each size, can be drawn from same diagram.

6. Upon lines  $m n$ ,  $o p$ ,  $q r$ , the addenda and depth of teeth are laid off, these lines being passed through the largest pitch diameter of gears. Lay off the adden-



dum, it being in these gears  $\frac{1}{4}$ ". This gives distance  $m n$ ,  $o p$ ,  $q r$  and  $u v$  equal to the working depth of teeth, in these gears  $\frac{1}{4}$ ". The addendum, of course, is measured from cone-pitch lines.

7. Draw lines  $O m$ ,  $O n$ ,  $O o$ ,  $O p$ ,  $O q$ ,  $O r$ . These lines give the height of teeth above cone-pitch lines and the working depth of teeth. The teeth become smaller as they approach  $O$  and would vanish entirely at  $O$ . It is quite as well never to have length of teeth, or face,  $m m'$  longer than one-third the distance  $m O$ , nor more than  $2\frac{1}{2}$  times the circular pitch.

8. Having decided upon the length of face, draw limiting lines  $m' n'$  perpendicular to  $i O$ ,  $q' r'$  perpendicular to  $k O$ , and so on.

The Whole Diameter of Bevel Gear Blanks is generally obtained by Measuring Drawings.

We have now the outline of section of gears through their axes. The distance  $m r$  is the *whole diameter* of pinion. The distance  $q o$  is the *whole diameter* of gear. In practice these diameters are obtained by *measuring* the drawing. The diameter of pinion is 3.45" and of gear 6.22". We also find angles by measuring drawing with a protractor. In the absence of a protractor templates can be cut to drawing. The angle formed by line  $m m'$  with  $a b$  is the angle of face of pinion, in this pinion  $59^{\circ} 11'$  or  $59\frac{1}{4}^{\circ}$  nearly. The lines  $q q'$  and  $g h$  give us angle of face of gear, for this gear  $22^{\circ} 19'$  or  $22\frac{1}{4}^{\circ}$  nearly. The angle formed by  $m n$  with  $a b$  is called the angle of edge of pinion, in our sketch  $26^{\circ} 34'$ , or about  $26\frac{1}{4}^{\circ}$ . The angle of edge of gear, line  $q r$  with  $g h$  is  $63^{\circ} 26'$ , or about  $63\frac{1}{4}^{\circ}$ . In turning blanks to these angles we place one arm of the protractor or template against *end of hub*, when trying angles of a blank. Some designers give angles from the axes of gears, but it is not convenient to try blanks in this way. The method we have given also comes right for angles as figured in compound rests.

When axes are at right angles, the sum of angles of edge in the two gears equals  $90^{\circ}$ , and the sums of angle of edge and face in each gear are alike.

The angle of axes remaining the same, all pairs of bevel gears of same ratio have the same angle of edge; all pairs of same ratio and of same numbers of teeth

have the same angles of both edges and faces independent of the pitch. Thus, in all pairs of bevel gears having one gear twice as large as the other, with axes at right angles, the angle of edge of large gear is  $63^{\circ} 26'$ , and the angle of edge of small gear is  $26^{\circ} 34'$ .

In all pairs of bevel gears with axes at right angles, one gear having 24 teeth and the other gear having 12 teeth, the angle of face of large gear is  $22^{\circ} 19'$ , and the angle of face of small gear is  $59^{\circ} 11'$ .

The following method of obtaining the whole diameter of bevel gears is sometimes preferred:

From  $k$  lay off, upon the cone-pitch line, a distance  $k w$ , equal to ten times the working depth of the teeth  $= 10 D'$ . Now add  $\frac{1}{10}$  of the shortest distance of  $w$  from the line  $g h$ , which is the perpendicular dotted line  $w x$ , to the outside pitch diameter of gear, and the sum will be the whole diameter of gear. In the same manner  $\frac{1}{10}$  of  $w y$ , added to the outside pitch diameter of pinion, gives the whole diameter of pinion.

Another method of obtaining Whole Diameter of Blanks.

## CHAPTER X.

## BEVEL GEARS—FORM AND SIZE OF TEETH—CUTTING TEETH.

To obtain data for teeth, we need only make drawing of section of one-half of each gear.

1. We first draw center lines  $A O$  and  $B O$ , Fig. 17, and the lines  $g h$  and  $c d$ , then gear blank lines as in Chapter IX.

Shape of teeth  
in bevel gears.

To obtain shape of teeth in bevel gears we do not lay them off on pitch circles the same as we would in spur gears. A line running from a point on cone-pitch line to center line of a bevel gear and perpendicular to the cone-pitch line, is the radius for circle upon which to draw outlines of teeth at this point.

2. Thus,  $A c$  is the *geometrical* pitch circle radius for large end of teeth, and  $A' c'$  the geometrical pitch radius for small end of teeth of wheel. To avoid confusion, we have transferred the distance  $A' c'$  to the line  $A c''$ .

3. For the pinion we have the geometrical pitch circle radius  $B c$  for large end of teeth, and the radius  $B' c'$  for small end of teeth. We have transferred the distance  $B' c'$  to the line  $B c'''$ .

4. About  $A$  draw the arc  $c n m$ , and upon it lay off spaces equal to the thickness of tooth at pitch line, and draw outlines of teeth as in previous chapters. That is, for single-curve teeth draw a semicircle upon radius  $A c$  and proceed as in Chapter III. For all bevel gears cut with a rotary disk-cutter (a common gear-cutter), single-curve teeth are preferable. Double-curve teeth can be drawn from instructions in Chapters VII. and VIII. We now have the shape of teeth at large end of gear. Repeat this operation with radius  $B c$  about  $B$ , and we have form of teeth at large end of pinion.

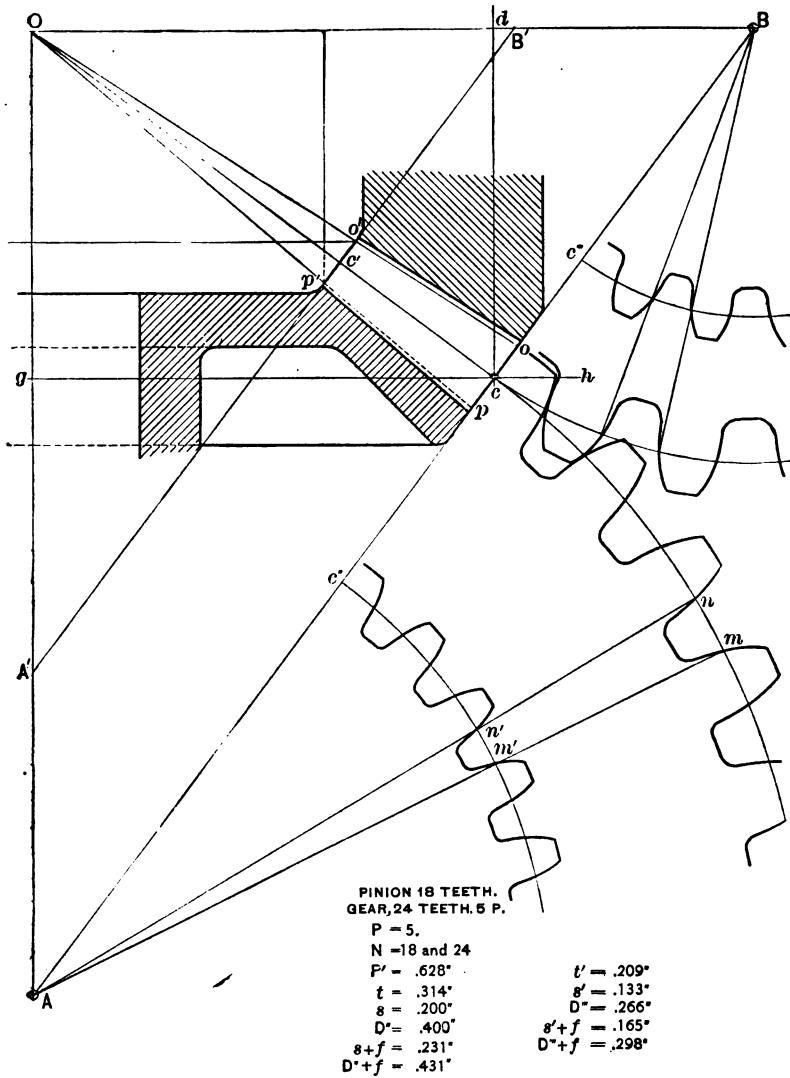


Fig. 17

BEVEL GEARS, FORM AND SIZE OF TEETH.



5. Upon arc of radius  $A' c'$  we get shape of teeth at small end of gear, and upon arc of radius  $B' c'$  we get shape of teeth at small end of pinion.

Sizes of teeth  
at small end  
by diagram.

6. The sizes of tooth parts at small end can be taken directly from diagram, or they can be calculated. By measuring the distance  $p' c'$  and  $c' o'$ , we obtain addendum and working depth of teeth at inside of the gears. The thickness of teeth at inside is given at  $m' n'$  by lines  $m n$  running to A, the distance  $m n$  being the thickness of tooth at outside. We designate tooth parts at inside by adding one more accent to the corresponding designation for tooth parts at outside, which are the same as those for spur gears. The tooth parts inside are denoted thus:  $t', s', D''', s' + f, D''' + f$ . All elements of the teeth run to the point O, where axes of gears meet. The sizes of tooth parts at small end can be calculated as follows: (These sizes bear the same ratio to the corresponding sizes at outside of gears that the distance  $O c'$  does to  $O c$ .)

Sizes of teeth  
at small end by  
calculation.

Dividing the distance  $O c'$ , which in our example is 2", by  $O c$ , which in this case is 3", we obtain  $\frac{2}{3}$  or .666 as the ratio. Multiplying the outside sizes by .666, we obtain the corresponding inside sizes (see Fig. 17). Thus, the thickness of teeth at outside being .314, we take  $\frac{2}{3}$  of .314" and obtain .209" as the thickness of teeth inside.

Cutting Bevel  
Gears with  
Rotary Cutters.

When cutting bevel gears with rotary cutters the angle of cutter-head is set the same as angle of working depth; thus: To cut the gear we have the cutter travel in the direction of line  $O p$ . The angle  $A O p$  is called the "Cutting Angle," being measured from the axis of gear. In the method here adopted the angle of face of pinion is the same as cutting angle of gear, and face angle of gear is the cutting angle of pinion; so that  $f$ , or clearance at bottom of spaces, is the same at inside of gear as  $f$  at the outside of gear.

Side Angle  
of Cutter-Head.

The cutter-head in some gear-cutting engines can be set at an angle sidewise as well as toward the work spindle.

In such engines, for cutting bevel gears, take cutter of right curve for teeth about one-third the length of tooth from large end; this gives teeth with sides not

enough curved at small end. In pinions it is sometimes necessary to file teeth at small ends; the cutter should, of course, be thin enough to go through space at small end. In these engines the spaces at large end of teeth are made wide enough by setting the cutter-head sidewise, two settings and two cuts being applied to each gear; but after the first gear the setting is changed once only for each gear. In most engines there is no provision for side angle, and the width of spaces is obtained by turning the work spindle and setting cutter out of center. When the cutting angle of gears is greater than  $45^\circ$  the teeth cut by the two engines are about alike; but when the cutting angle is less than  $45^\circ$ , the same cutter will make the addendum of teeth more rounding in the cutting engine with no side angle than it will in the cutting engine with side angle. In such cases a corresponding allowance should be made in the shaping of cutters. In ordering bevel-gear cutters it should be stated which kind of engine the cutters are to be used in. The Universal Milling Machine can be used for either side angle or rotation of work spindle.

Different Cutter for Machines without Side Angle.

Universal Milling Machines can be used both ways.

We can plane bevel gears up to 16" diameter, theoretically exact, each gear being an original one.

We can plane Accurate Bevel Gears.



## CHAPTER XI.

## WORM WHEELS—SIZING BLANKS OF 32 TEETH AND OVER.

Worm.

A WORM is a screw made to mesh with the teeth of a wheel called a *worm-wheel*. As implied at the end of Chapter IV., a section of a worm through its axis is, in outline, the same as a rack of corresponding pitch. This outline can be made either to mesh with single or double curve gear teeth; but worms are usually made for single curve, because, the sides of involute rack teeth being straight (see Chapter IV.), the tool for cutting worm-thread is more easily made. The thread-tool is not usually rounded for giving fillets at bottom of worm-thread.

The rules for circular pitch apply in the size of tooth parts and diameter of pitch-circle of worm-wheel.

Pitch of Worm.

The pitch of a worm or screw is usually given in a way different from the pitch of a gear, viz.: in number of threads to one inch of the length of the worm or screw. Thus, if we say a worm is 2 pitch we mean 2 threads to the inch, or the worm makes two turns to advance the thread one inch. But a worm may be double-threaded, triple-threaded, and so on.

Lead of a Worm-Thread.

To avoid misunderstanding it is better always to call the advance of the worm thread the *lead*. Thus, a worm-thread that advances one inch in one turn we call one-inch *lead* in one turn. A single-thread worm 4 to 1" is  $\frac{1}{4}$ " lead. We apply the term pitch to the actual distance between the threads or teeth, as in previous chapters. In single-thread worms the lead and the pitch are alike. If we have to make a worm and wheel so many threads to one inch, we first *divide 1" by the number of threads to one inch*, and the quotient gives us the *circular pitch*. Hence, the wheel in Fig. 19 is  $\frac{1}{2}$ "

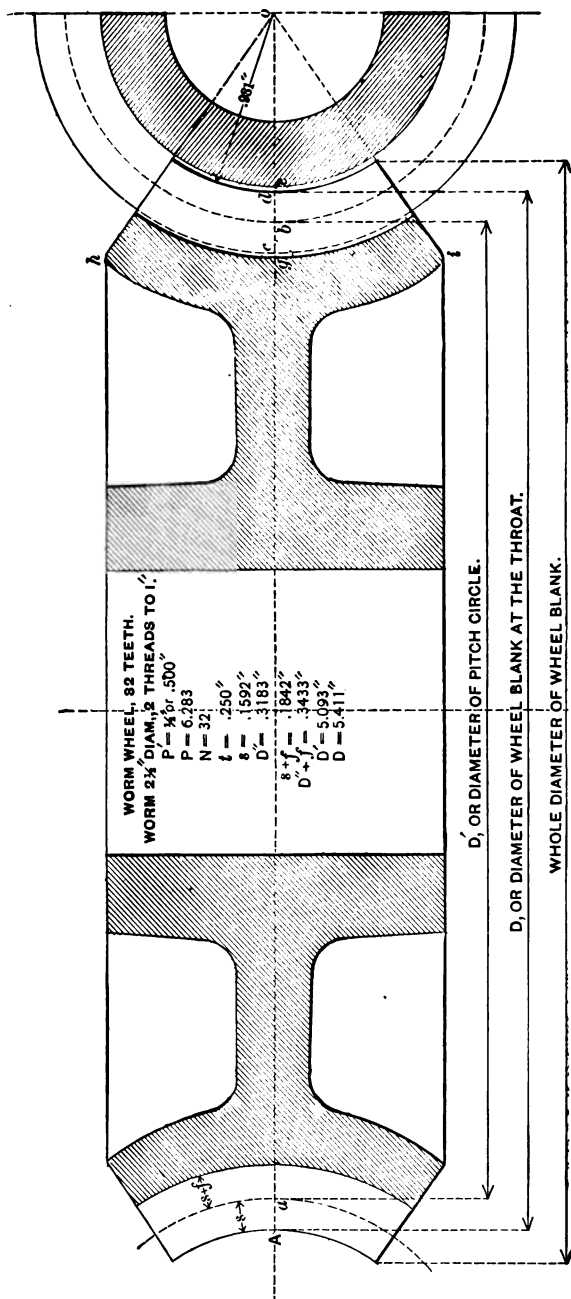
Linear Pitch.

circular pitch. The term *linear pitch* expresses ex-



FIG. 18.—WORM AND WORM-WHEEL.

The thread of Worm is left-handed ; Worm is single-threaded.



actly what is meant by circular pitch. Linear pitch has the advantage of being an exact use of language when applied to worms and racks. The number of threads to one inch linear, is the reciprocal of the linear pitch.

Multiply 3.1416 by the number of threads to one inch, and the product will be the diametral pitch of the worm-wheel. Thus, we would say of a double-thread worm advancing 1" in  $1\frac{1}{2}$  turns that:

Lead =  $\frac{2}{3}$ " or .75". Linear pitch or  $P' = \frac{3}{2}$ " or .375". Drawing of Worm and Worm-wheel.  
Diametral pitch or  $P = 8.377$ . See table of tooth parts.

To make drawing of worm and wheel we obtain data as in circular pitch.

1. Draw center line A O and upon it space off the distance  $a b$  equal to the diameter of pitch-circle.
2. On each side of these two points lay off the distance  $s$ , or the usual addendum =  $\frac{1}{P}$ ", as  $b c$  and  $b d$ .
3. From  $c$  lay off the distance  $c O$  equal to the radius of the worm. The diameter of a worm is generally four or five times the circular pitch.
4. Lay off the distances  $c g$  and  $d e$  each equal to  $f$ , or the usual clearance at bottom of tooth space.
5. Through  $c$  and  $e$  draw circles about O. These represent the whole diameter of worm and the diameter at bottom of worm-thread.
6. Draw  $h O$  and  $i O$  at an angle of  $30^\circ$  to  $45^\circ$  with A O. These lines give width of face of worm-wheel.
7. Through  $g$  and  $d$  draw arcs about O, ending in  $h O$  and  $i O$ .

This operation repeated at  $a$  completes the outline of worm-wheel. For 32 teeth and more, the addendum diameter, or D, should be taken at the throat or smallest diameter of wheel, as in Fig. 19. *Measure sketch for whole diameter of wheel-blank.*

The foregoing instructions and sketch are for cases where the teeth of the wheels are finished with a hob. Teeth of Wheels finished with Hob.

A HOB is shown in Fig. 20, being a steel piece threaded with the same tool that threads the worm, then grooved to make teeth for cutting, and hardened. Hob.

The whole diameter of hob should be  $2f$ , or twice the clearance larger than the worm. The outer cor- Proportions of Hob.

ners of hob-thread can be rounded down as far as the clearance distance. The width at top of the hob-thread before rounding should be  $.31$  of the linear, or circular pitch  $= .31P'$ . The whole depth of thread should be the ordinary working depth plus twice the clearance  $= D'' + 2f$ . The diameter at bottom of hob-thread should be the same as diameter at bottom of worm-thread. For thread-tool and worm-thread see end of Chapter IV. The thickness of cutter for grooving hob can be about  $\frac{1}{2}$  the width of thread at top plus  $\frac{1}{8}'' = .3\frac{1}{2}P' + \frac{1}{8}''$ . The width of lands at the bottom can be



FIG. 20.—HOB.  
Right-handed thread.

about the depth of thread plus  $\frac{1}{8}'' = D'' + 2f + \frac{1}{8}''$ . The grooves are usually cut with a round-edge cutter, the parallel part of cutter just reaching the bottom of thread, making the half-round bottom of grooves below the bottom of thread. In most hobs the teeth are not relieved between the grooves, because it takes considerable time to file or cut the relief. The bur thrown out by the grooving cutter can be left on if it is not too rough. The bur will make the hob cut faster. Alternate grooves can be cut in opposite direction, so that worm will not cut

faster on one side than on the other. If many wheels are to be finished with the hob, the teeth should be relieved.

The teeth of the wheel are first cut as nearly to the finished form as practicable; the hob and worm-wheel are mounted upon shafts and hob placed in mesh as in Fig. 18. The hob is now made to rotate, and is dropped deeper into the wheel at each revolution of the wheel until teeth are finished. The hob generally drives the worm-wheel during this operation. The Universal Milling Machine is very convenient for doing this work, and with it the distance between axes of worm and wheel can be readily noted. Where a great many wheels are manufactured special machines have been made for driving wheels independent of hob. The object of hobbing a wheel is to get more bearing surface of the teeth upon worm-thread. The worm-wheels, Figs. 18 and 26, were hobbled. By hobbing we produce outline of teeth something like the thread of a nut.

How to use the Hob.

Universal Milling Machine used in Hobbing.

Why a Wheel is Hobbled.

If we make the diameter of a worm-wheel blank, that is to have less than 30 teeth, by the common rules for sizing blanks, and finish the teeth with a hob, we shall find the flanks of teeth near the bottom to be undercut or hollowing. This is caused by the interference spoken of in Chapter VI. Thirty teeth was there given as a limit, which will be right when teeth are made to circle arcs. With pressure angle  $75\frac{1}{2}^{\circ}$ , and rack-teeth with usual addendum, this interference of rack-teeth with flanks of gear-teeth commences at 31 teeth ( $31\frac{1}{6}$  geometrically), and interferes with nearly the whole flank in wheel of 12 teeth.

Worm-Wheel Blanks with Less than 30 Teeth.

Interference of Thread and Flank.

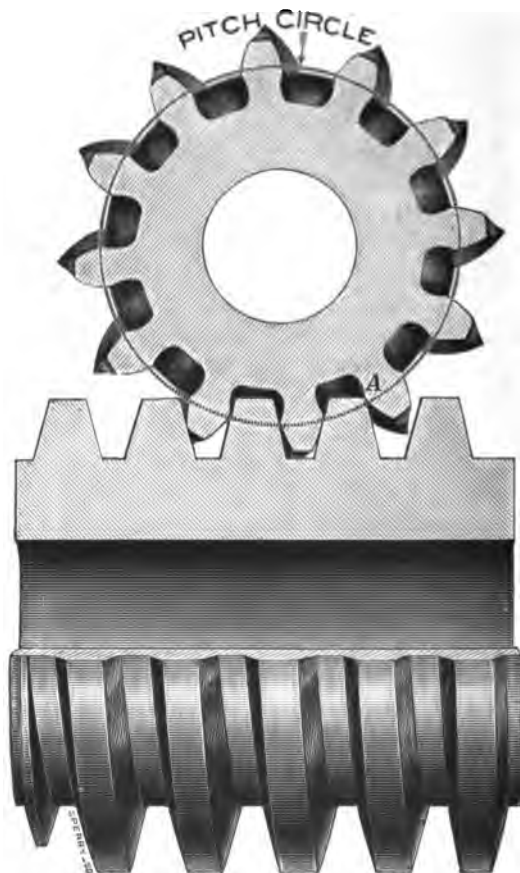
In Fig 21 the blank for worm-wheel of 12 teeth was sized by the same rule as given for Fig. 19. The wheel and worm are sectioned to show shape of teeth at the mid-plane of wheel. The flanks of teeth are undercut by the hob. The worm-thread does not have a good bearing on flanks inside of A, the bearing being that of a corner against a surface.

Fig. 21.

In Fig 22 the blank for wheel was sized so that pitch-circle comes midway between outermost part of teeth and innermost point obtained by worm thread.

Fig. 22.



**Fig. 21.**

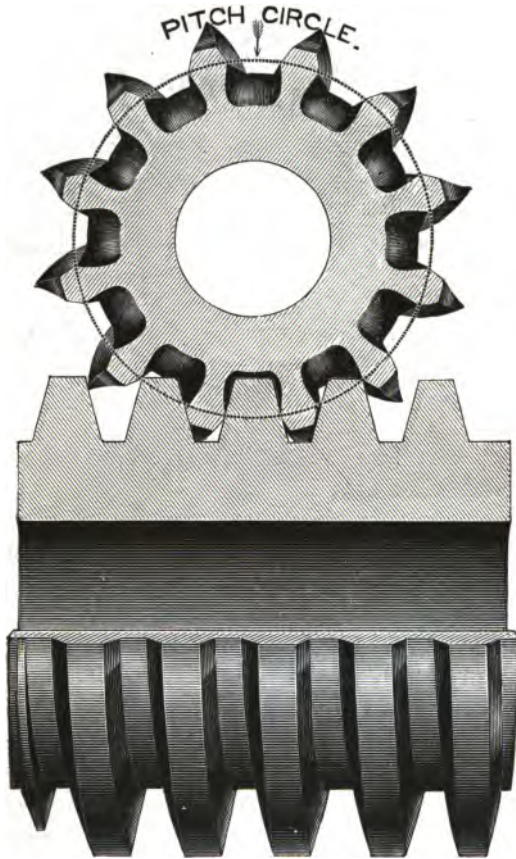


Fig. 22.

This rule for sizing worm-wheel blanks has been in use to some extent. The hob has cut away flanks of teeth still more than in Fig. 21. The pitch-circle in Fig. 22 is the same diameter as the pitch-circle in Fig. 21. The same hob was used for both wheels. The flanks in this wheel are so much undercut as to materially lessen the bearing surface of teeth and worm-thread.

Interference  
Avoided.

In Chapter VI. the interference of teeth in high-numbered gears and racks with flanks of 12 teeth was remedied by rounding off the addenda. Although it would be more systematic to round off the threads of a worm, making them, like rack-teeth, to mesh with interchangeable gears, yet this has not generally been done, because it is easier to make a worm-thread tool with straight sides.

Instead of cutting away the addenda of worm-thread, we can avoid the interference with flanks of wheels having less than 30 teeth by making wheel blanks larger.

Fig. 23.

The flanks of wheel in Fig. 23 are not undercut, because the diameter of wheel is so large that there is hardly any tooth inside the pitch-circle. The pitch-circle in Fig. 23 is the same size as pitch-circles in Figs. 21 and 22. This wheel was sized by the following rule: Multiply the *pitch diameter* of the wheel by .937, and add to the product four times the addendum ( $4s$ ); the sum will be the diameter for the blank at the throat or small part. To get the whole diameter, make a sketch with diameter of throat to the foregoing rule and measure the sketch.

Diameter at  
Throat to Avoid  
Interference.

It is impractical to hob a wheel of 12 to about 16 or 18 teeth when blank is sized by this rule, unless the wheel is driven by independent mechanism and not by the hob. The diameter across the outermost parts of teeth, as at A B, is considerably less than the largest diameter of wheel before it was hobbled.

In general it is well to size all blanks, as by page 45 and Figs. 19 and 21, when the wheels are to be hobbled. Of course, if the wheel is to be hobbled the

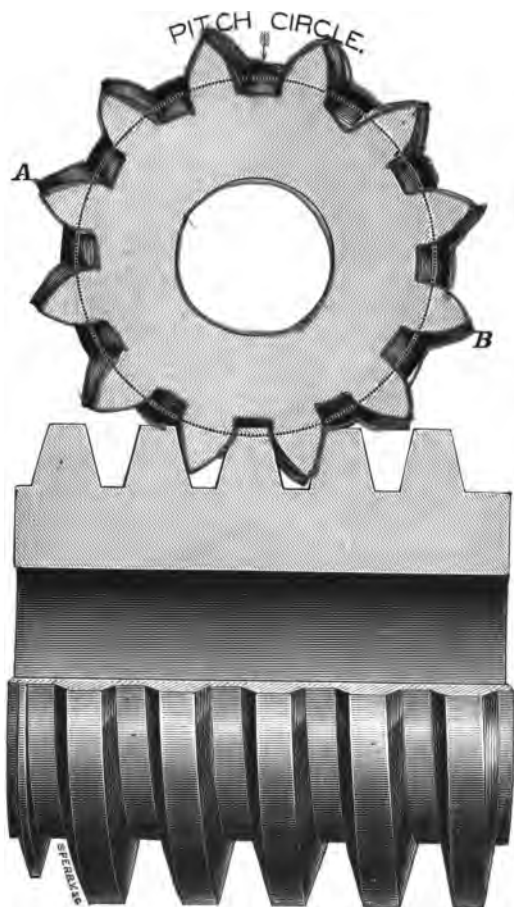


Fig. 23.

cutter should be thin enough to leave stock for finishing. The spaces can be cut the full depth, the cutter being dropped in.

To get angle of worm-thread, it is best to apply protractor directly to the thread, as computing the angle affords but little help. Set gear cutter-head as near the angle as can be seen from trial with protractor upon thread; cut a few teeth; try in worm. Generally the cutter-head has to be changed before the worm will take the right position.

When worm-wheels are not hobbled it is better to turn blanks like a spur-wheel. Little is gained by having wheels curved to fit worm unless teeth are finished with a hob. The teeth can be cut in a straight path diagonally across face of blank, to fit angle of worm-thread, as in Figs. 24 and 27.

Blank Like a  
Spur-Wheel.

Wheels for  
Gear-Cutting  
Machines.

For dividing wheels to gear-cutting engines the blanks are turned like a spur-wheel and a cutter about  $\frac{1}{16}$ " larger diameter than the worm, is dropped in, as in Figs. 25 and 28, and the worm-thread is slightly rounded at the outer corners. The radius for rounding thread can be  $\frac{1}{4}$  the width of thread at the top.

Some mechanics prefer to make dividing wheels in two parts, joined in a plane perpendicular to axis, hob teeth; then turn one part round upon the other, match teeth and fasten parts together in the new position, and hob again with a view to eliminate errors.

With an accurate cutting engine we have found wheels like Figs. 25 and 28, not hobbled, every way satisfactory. Dividing wheels of 2 feet diameter and less are generally made without arms, the part between hub and rim being a solid web. As to the different wheels, Figs. 26, 27 and 28, when worm is in right position at the start, the life-time of Fig. 26, under heavy and continuous work, will be the longest.

Figures 26, 27  
and 28.

Fig. 27 can be run in mesh with a gear or a rack as well as with a worm. At the start the contact of teeth in this wheel upon worm-thread is in points only; yet such wheels have been many years successfully used in elevators.

Fig. 28 is a neat-looking wheel. In gear-cutting



**Fig. 24.**

**Worm-wheel with teeth cut in a straight path diagonally across face.  
Worm is double-threaded.**



Fig. 25.

Worm and Worm-Wheel, for Gear-cutting Engine.



Fig. 26.



Fig. 27.



Fig. 28.



engines where the workman has occasion to turn the work spindle by hand, it is not so rough to take hold of as Figs. 26 and 27. The teeth are less liable to injury than the teeth of Figs. 26 and 27.

Some designers prefer to take off the outermost part of teeth in wheels (Figs. 18 and 26), as shown in these two figures, and not leave them sharp, as in Fig. 19.

We do not know that this serves any purpose except a matter of looks.

## CHAPTER XII.

# SIZING GEARS WHEN THE DISTANCE BETWEEN CENTERS AND THE RATIOS OF SPEEDS ARE FIXED—GENERAL REMARKS—WIDTH OF FACE OF SPUR GEARS—SPEED OF GEAR CUTTERS—TABLE OF TOOTH PARTS.

Let us suppose that we have two shafts 14" apart, center to center, and wish to connect them by gears so that they will have speed ratio 6 to 1. We add the 6 and 1 together, and divide 14" by the sum and get 2" for a quotient; this 2", multiplied by 6, gives us the radius of pitch circle of large wheel = 12". In the same manner we get 2" as radius of pitch circle of small wheel. Doubling the radius of each gear, we obtain 24" and 4" as the pitch diameters of the two wheels. The two numbers that form a ratio are called the terms of the ratio. We have now the rule for obtaining pitch-circle diameter of two wheels of a given ratio to connect shafts a given distance apart:

*Divide the center distance by the sum of the terms of the ratio; find the product of twice the quotient by each term separately, and the two products will be the pitch diameters of the two wheels.*

Center distance and Ratio fixed.  
Rule for Diameter of Pitch Circles.

It is well to give special attention to learning the rules for sizing blanks and teeth; these are much oftener needed than the method of forming tooth outlines.

A blank  $1\frac{1}{2}$ " diameter is to have 16 teeth: what will the pitch be? What will be the diameter of the pitch circle? See Chapter V.

A good practice will be to compute a table of tooth parts. The work can be compared with the tables pages 64, 67.

In computing it is well to take  $\pi$  to more than four places,  $\pi$  to nine places = 3.141592653.  $\frac{1}{\pi}$  to nine places = .318309886.

There is no such thing as pure rolling contact in teeth of wheels; they always rub, and, in time, will wear themselves out of shape and may become noisy.

Bevel gears, when correctly formed, run smoother than spur gears of same diameter and pitch, because the teeth continue in contact longer than the teeth of spur gears. For this reason annular gears run smoother than either bevel or spur gears.

Sometimes gears have to be cut a little deeper than designed, in order to run easily on their shafts. If any departure is made in ratio of pitch diameters it is better to have the driving gear the larger, that is, cut the follower smaller. For wheels coarser than eight diametral pitch (8 P), it is generally better to cut twice around, when accurate work is wanted, also for large wheels, as the expansion of parts from heat often causes inaccurate work when cut but once around. There is not so much trouble from heat in plain or web gears as in arm gears.

Width of Spur  
Gear faces.

The width of cast-iron gear faces for general purposes can be made to the following rule:

*Divide 8 by the diametral pitch and add  $\frac{1}{4}$ " to the quotient; the sum will be width of face for the pitch required.*

Example: What width of face for gear 4 P? Dividing 8 by 4 and adding  $\frac{1}{4}$ " we obtain  $2\frac{1}{4}$ ", for width of face. For change gears on lathes, where it is desirable not to have face very wide, the following rule can be used:

*Divide 4 by the diametral pitch and add  $\frac{1}{2}$ ".*

By the latter rule a 4 P change gear would have but  $1\frac{1}{2}$ " face.

Speed of Gear  
Cutters.

The speed of gear cutters is subject to so many conditions that definite rules cannot be given. We append a table of average speeds. A coarse pitch cutter for pinion, 12 teeth, would usually be run slower than a cutter for a large gear of same pitch.

TABLE OF AVERAGE SPEEDS FOR GEAR-CUTTERS.

Diametral Pitch of Cutter.	Diameter of Cutter.	Turns per minute cutting Cast Iron.	Turns per minute cutting Wrought Iron and Steel.	Feed to One Turn of Cutter in Cast Iron.	Feed to One Turn of Cutter in Wrought Iron and Steel.	Feed per minute in Cast Iron.	Feed per minute in Wrought Iron and Steel.
2	5 in.	24	18	.025 in.	.011 in.	.60 in.	.20 in.
2½	4½ "	30	24	.028 "	.013 "	.84 "	.31 "
3	3½ "	36	28	.031 "	.015 "	1.12 "	.42 "
4	3 "	42	32	.034 "	.017 "	1.43 "	.54 "
5	3½ "	50	40	.037 "	.019 "	1.85 "	.76 "
6	2½ "	75	55	.030 "	.016 "	2.25 "	.88 "
7	2½ "	85	65	.032 "	.018 "	2.72 "	1.17 "
8	2½ "	95	75	.034 "	.020 "	3.23 "	1.50 "
10	2½ "	125	90	.026 "	.014 "	3.25 "	1.26 "
12	2 "	135	100	.027 "	.017 "	3.64 "	1.70 "
20	1½ "	145	115	.029 "	.021 "	4.20 "	2.41 "
32	1½ "	160	135	.031 "	.025 "	4.96 "	3.37 "

In brass the speed of gear-cutters can be twice as <sup>Speed in</sup> fast as in cast iron. Clock-makers and those making a <sup>Brass.</sup> specialty of brass gears exceed this rate even. A 12 P cutter has been run 1,200 (twelve hundred) turns a minute in bronze. A 32 P cutter has been run 7,000 (seven thousand) turns a minute in soft brass.

In cutting 5 P cast-iron gears, 75 teeth, a No. 1, 6 P <sup>Examples</sup> cutter was run 136 (one hundred and thirty-six) turns <sup>from Practice.</sup> a minute, roughing the spaces out the full 5 P depth ; the teeth were then finished with a 5 P cutter, running 208 (two hundred and eight) turns a minute, feeding by hand. The cutter stood well, but, of course, the cast iron was quite soft. A 4 P cutter has finished teeth at one cut, in cast-iron gears, 86 teeth, running 48 (forty-eight) turns a minute and feeding  $\frac{1}{16}$ " at one turn, or 3 in. in a minute.

Hence, while it is generally safe to run cutters as in the table, yet when many gears are to be cut it is well to see if cutters will stand a higher speed and more feed.

In gears coarser than 4 P it is more economical to first cut the full depth with a stocking cutter and then finish with a gear cutter. This stocking cutter is made

on the principle of a circular splitting saw for wood. The teeth, however, are not set ; but side relief is obtained by making sides of cutter blank hollowing. The shape of stocking cutter can be same as bottom of spaces in a 12-tooth gear, and the thickness of cutter can be  $\frac{1}{3}$  of the circular pitch.

Keep Cutters  
sharp.

The matter of keeping cutters sharp is so important that it has sometimes been found best to have the workman grind them at stated times, and not wait until he can see that the cutters are dull. Thus, have him grind every two hours or after cutting a stated number of gears. Cutters of the style that can be ground upon their tooth faces without changing form are rapidly destroyed if allowed to run after they are dull. Cutters are oftener wasted by trying to cut with them when they are dull than by too much grinding. Grind the faces radial with a free cutting wheel. Do not let the wheel become glazed, as this will draw the temper of the cutter.

In Chapter VI. was given a series of cutters for cutting gears having 12 teeth and more. Thus, it was there implied that any gear of same pitch, having 135 teeth, 136 teeth, and so on up to the largest gears, and, also, a rack, could be cut with one cutter. If this cutter is 4 P, we would cut with it all 4 P gears, having 135 teeth or more, and we would also cut with it a 4 P rack. Now, instead of always referring to a cutter by the number of teeth in gears it is designed to cut, it has been found convenient to designate it by a letter or by a number. Thus, we call a cutter of 4 P, made to cut gears 135 teeth to a rack, inclusive, No. 1, 4 P.

We have adopted numbers for designating involute

Involute Gear  
Cutters.

gear-cutters as in the following table:

No. 1 will cut wheels from 135 teeth to a rack inclusive.							
"	2	"	"	55	"	134 teeth	"
"	3	"	"	35	"	54	"
"	4	"	"	26	"	34	"
"	5	"	"	21	"	25	"
"	6	"	"	17	"	20	"
"	7	"	"	14	"	16	"
"	8	"	"	12	"	13	"

By this plan it takes eight cutters to cut all gears having twelve teeth and over, of any one pitch.

Thus it takes eight cutters to cut all involute 4 P gears having twelve teeth and more. It takes eight other cutters to cut all involute gears of 5 P, having 12 teeth and more. A No. 8, 5 P cutter cuts only 5 P gears having 12 and 13 teeth. A No. 6, 10 P cutter cuts only 10 P gears having 17, 18, 19 and 20 teeth. On each cutter is stamped the number of teeth at the limits of its range, as well as the number of the cutter. The number of the cutter relates only to the number of teeth in gears that the cutter is made for.

In ordering cutters for involute spur-gears two things must be given :

1. *Either the number of teeth to be cut in the gear or the number of the cutter, as given in the foregoing table.* How to order  
Involute Cut-  
ters.

2. *Either the pitch of the gear or the diameter and number of teeth to be cut in the gear.*

If 25 teeth are to be cut in a 6 P involute gear, the cutter will be No. 5, 6 P, which cuts all 6 P gears from 21 to 25 teeth inclusive. If it is desired to cut gears from 15 to 25 teeth, three cutters will be needed, No. 5, No. 6 and No. 7 of the pitch required. If the pitch is 8 and gears 15 to 25 teeth are to be cut, the cutters should be No. 5, 8 P, No. 6, 8 P, and No. 7, 8 P.

For each pitch of epicycloidal, or double-curve gears, Epicycloidal  
or Double-  
curve Cutters. 24 cutters are made. In coarse-pitch gears, the variation in the shape of spaces between gears of consecutive-numbered teeth is greater than in fine-pitch gears.

Having a set of cutters for each pitch, to consist of so large a number as 24, has been established because double-curve teeth have generally been preferred in coarse-pitch gears.

Double-curve gear-cutters are designated by letters instead of by numbers; this is to avoid confusion in ordering.

Following is the list of epicycloidal or double-curve gear-cutters:—

Table of Epi-  
cycloidal or  
Double-curve  
Gear Cutters.

Cutter A cuts 12 teeth.	Cutter M cuts 27 to 29 teeth.
" B " 13 "	" N " 30 " 33 "
" C " 14 "	" O " 34 " 37 "
" D " 15 "	" P " 38 " 42 "
" E " 16 "	" Q " 43 " 49 "
" F " 17 "	" R " 50 " 59 "
" G " 18 "	" S " 60 " 74 "
" H " 19 "	" T " 75 " 99 "
" I " 20 "	" U " 100 " 149 "
" J " 21 to 22 "	" V " 150 " 249 "
" K " 23 to 24 "	" W " 250 " Rack.
" L " 24 to 26 "	" X " Rack.

A cutter that cuts more than one gear is made of proper form for the smallest gear in its range. Thus, cutter J for 21 to 22 teeth is correct for 21 teeth; cutter S for 60 to 74 teeth is correct for 60 teeth, and so on.

How to order  
Epicycloidal  
Cutters.

In ordering epicycloidal gear-cutters designate the letter of the cutter as in the foregoing table, also either give the pitch or give data that will enable us to determine the pitch, the same as directed for involute cutters.

More care is required in making and adjusting epicycloidal gears than in making involute gears.

How to order  
Bevel Gear  
Cutters.

In ordering bevel gear cutters three things must be given :

1. *The number of teeth in each gear.*
2. *Either the pitch of gears or the largest pitch diameter of each gear; see Fig. 16.*
3. *The length of tooth face.*

If the shafts are not to run at right angles, it should be so stated, and the angle given. Involute cutters only are used for cutting bevel gears. No attempt should be made to cut epicycloidal tooth bevel gears.

How to order  
Worm - gear  
Cutters.

In ordering worm-wheel cutters, three things must be given :

1. *Number of teeth in the wheel.*
2. *Pitch of the worm; see Chapter XI.*
3. *Whole diameter of worm.*

In any order connected with gears or gear-cutters, when the word "Diameter" occurs, we usually understand that the *pitch diameter* is meant. When the *whole* diameter of a gear is meant it should be plainly written. Care in giving an order often saves the delay of asking further instructions. An order for one gear-cutter to cut from 25 to 30 teeth cannot be filled, because it takes two cutters of involute form to cut from 25 to 30 teeth, and three cutters of epicycloidal form to cut from 25 to 30 teeth.

Sheet zinc is convenient to sketch gears upon, and also for making templets. Before making sketch, it is well to give the zinc a dark coating with the following mixture: Dissolve 1 ounce of sulphate of copper (blue vitriol) in about 4 ounces of water, and add about one-half teaspoonful of nitric acid. Apply a thin coating with a piece of waste.

This mixture will give a thin coating of copper to iron or steel, but the work should then be rubbed dry. Care should be taken not to leave the mixture where it is not wanted, as it rusts iron and steel.



## GEAR WHEELS.

TABLE OF TOOTH PARTS—CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1}{2}$ "	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P'	P"	P	t	s	D"	s+f	D"+f	P'×.31	P'×.35
2	$\frac{1}{2}$	1.5708	1.0000	.6366	1.2732	.7366	1.3732	.6200	.6700
$1\frac{1}{8}$	$\frac{3}{8}$	1.6755	.9375	.5968	1.1937	.6906	1.2874	.5813	.6281
$1\frac{1}{4}$	$\frac{4}{4}$	1.7952	.8750	.5570	1.1141	.6445	1.2016	.5425	.5863
$1\frac{3}{8}$	$\frac{5}{8}$	1.9333	.8125	.5173	1.0345	.5985	1.1158	.5038	.5444
$1\frac{1}{2}$	$\frac{3}{2}$	2.0944	.7500	.4775	.9549	.5525	1.0299	.4650	.5025
$1\frac{7}{8}$	$\frac{1}{2}$	2.1855	.7187	.4576	.9151	.5294	.9870	.4456	.4816
$1\frac{3}{4}$	$\frac{3}{4}$	2.2848	.6875	.4377	.8754	.5064	.9441	.4262	.4606
$1\frac{5}{8}$	$\frac{3}{4}$	2.3936	.6562	.4178	.8356	.4834	.9012	.4069	.4397
$1\frac{1}{2}$	$\frac{4}{4}$	2.5133	.6250	.3979	.7958	.4604	.8583	.3875	.4188
$1\frac{3}{8}$	$\frac{1}{2}$	2.6456	.5937	.3780	.7560	.4374	.8156	.3681	.3978
$1\frac{1}{8}$	$\frac{3}{8}$	2.7925	.5625	.3581	.7162	.4143	.7724	.3488	.3769
$1\frac{1}{8}$	$\frac{1}{2}$	2.9568	.5312	.3382	.6764	.3913	.7295	.3294	.3559
1	1	3.1416	.5000	.3183	.6366	.3683	.6866	.3100	.3350
$1\frac{1}{8}$	$1\frac{1}{8}$	3.3510	.4687	.2984	.5968	.3453	.6437	.2906	.3141
$\frac{7}{8}$	$1\frac{1}{4}$	3.5904	.4375	.2785	.5570	.3223	.6007	.2713	.2931
$1\frac{3}{8}$	$1\frac{3}{8}$	3.8666	.4062	.2586	.5173	.2993	.5579	.2519	.2722
$\frac{3}{4}$	$1\frac{1}{2}$	4.1888	.3750	.2387	.4775	.2762	.5150	.2325	.2513
$1\frac{1}{8}$	$1\frac{5}{8}$	4.5696	.3437	.2189	.4377	.2532	.4720	.2131	.2303
$\frac{3}{8}$	$1\frac{1}{2}$	4.7124	.3333	.2122	.4244	.2455	.4577	.2066	.2233

TABLE OF TOOTH PARTS.—*Continued.*

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1}{P}$ .	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P'	$\frac{1}{P''}$	P	t	s	D''	s+f	D'+f	P'×.81	P'×.835
$\frac{5}{8}$	$1\frac{3}{4}$	5.0265	.3125	.1989	.3979	.2301	.4291	.1938	.2094
$\frac{1}{8}$	$1\frac{1}{2}$	5.5851	.2812	.1790	.3581	.2071	.3862	.1744	.1884
$\frac{1}{2}$	2	6.2832	.2500	.1592	.3183	.1842	.3433	.1550	.1675
$\frac{1}{8}$	$2\frac{1}{4}$	7.1808	.2187	.1393	.2785	.1611	.3003	.1356	.1466
$\frac{3}{8}$	$2\frac{1}{2}$	7.8540	.2000	.1273	.2546	.1473	.2746	.1240	.1340
$\frac{3}{8}$	$2\frac{3}{4}$	8.3776	.1875	.1194	.2387	.1381	.2575	.1163	.1256
$\frac{1}{8}$	3	9.4248	.1666	.1061	.2122	.1228	.2289	.1033	.1117
$\frac{5}{16}$	$3\frac{1}{4}$	10.0531	.1562	.0995	.1989	.1151	.2146	.0969	.1047
$\frac{7}{16}$	$3\frac{1}{2}$	10.9956	.1429	.0909	.1819	.1052	.1962	.0886	.0957
$\frac{1}{4}$	4	12.5664	.1250	.0796	.1591	.0921	.1716	.0775	.0838
$\frac{3}{16}$	$4\frac{1}{4}$	14.1372	.1111	.0707	.1415	.0818	.1526	.0689	.0744
$\frac{1}{8}$	5	15.7080	.1000	.0637	.1273	.0737	.1373	.0620	.0670
$\frac{5}{16}$	$5\frac{1}{4}$	16.7552	.0937	.0597	.1194	.0690	.1287	.0581	.0628
$\frac{1}{8}$	6	18.8496	.0833	.0531	.1061	.0614	.1144	.0517	.0558
$\frac{1}{4}$	7	21.9911	.0714	.0455	.0910	.0526	.0981	.0443	.0479
$\frac{1}{8}$	8	25.1327	.0625	.0398	.0796	.0460	.0858	.0388	.0419
$\frac{1}{8}$	9	28.2743	.0555	.0354	.0707	.0409	.0763	.0344	.0372
$\frac{1}{10}$	10	31.4159	.0500	.0318	.0637	.0368	.0687	.0310	.0335
$\frac{1}{16}$	16	50.2655	.0312	.0199	.0398	.0230	.0429	.0194	.0209

## GEAR WHEELS.

TABLE OF TOOTH PARTS—DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1}{P}$ .	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P	P'	t	s	D''	s+f.	D''+f.
$\frac{1}{2}$	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
$\frac{3}{4}$	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
$1\frac{1}{2}$	2.5133	1.2566	.8000	1.6000	.9257	1.7257
$1\frac{3}{4}$	2.0944	1.0472	.6666	1.3333	.7714	1.4381
$1\frac{1}{2}$	1.7952	.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
$2\frac{1}{4}$	1.3963	.6981	.4444	.8888	.5143	.9587
$2\frac{1}{2}$	1.2566	.6283	.4000	.8000	.4628	.8628
$2\frac{3}{4}$	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
$3\frac{1}{2}$	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	.0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541

TABLE OF TOOTH PARTS—*Continued.*

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1}{P}$ .	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P.	P'.	<i>t.</i>	<i>s.</i>	D''.	<i>s</i> + <i>f.</i>	D'' + <i>f.</i>
15	.2094	.1047	.0666	.1333	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.058	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0643	.1198
19	.1653	.0827	.0526	.1053	.0609	.1135
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
34	.0924	.0462	.0294	.0588	.0340	.0634
36	.0873	.0436	.0278	.0555	.0321	.0599
38	.0827	.0413	.0263	.0526	.0304	.0568
40	.0785	.0393	.0250	.0500	.0289	.0539
42	.0748	.0374	.0238	.0476	.0275	.0514
44	.0714	.0357	.0227	.0455	.0263	.0490
46	.0683	.0341	.0217	.0435	.0252	.0469
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431
56	.0561	.0280	.0178	.0357	.0207	.0385
60	.0524	.0262	.0166	.0333	.0193	.0360



# PART II.

## CHAPTER I.

### TANGENT OF ARC AND ANGLE.

In PART II. we shall show how to calculate some of the functions of a right-angle triangle from a table of circular functions, the application of these calculations in some chapters of PART I. and in sizing blanks and cutting teeth of spiral gears, the selection of cutters for spiral gears, the application of continued fractions to some problems in gear wheels and cutting odd screw-threads, etc., etc. Subjects to be explained.

A *Function* is a quantity that depends upon another quantity for its value. Thus the amount a workman earns is a function of the time he has worked and of his wages per hour. Function defined.

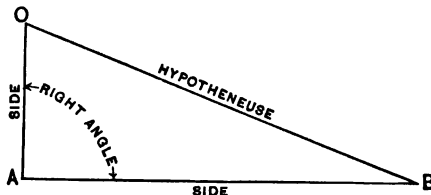


Fig. 29

In any right-angle triangle, O A B, we shall, for convenience, call the two lines that form the right angle O A B the *sides*, instead of base and perpendicular. Thus O A B, being the right angle we call the line O A a side, and the line A B a side also. Right-angle Triangle.

When we speak of the angle A O B, we call the line O A the *side adjacent*. When we are speaking of the angle A B O we call the line A B the side adjacent. side adjacent. The line opposite the right angle is the *hypotenuse*. Hypotenuse.

**Tangent.**

*The Tangent* of an arc is the line that touches it at one extremity and is terminated by a line drawn from the center through the other extremity. The tangent is always *outside* the arc and is also perpendicular to the radius which meets it at the point of tangency.

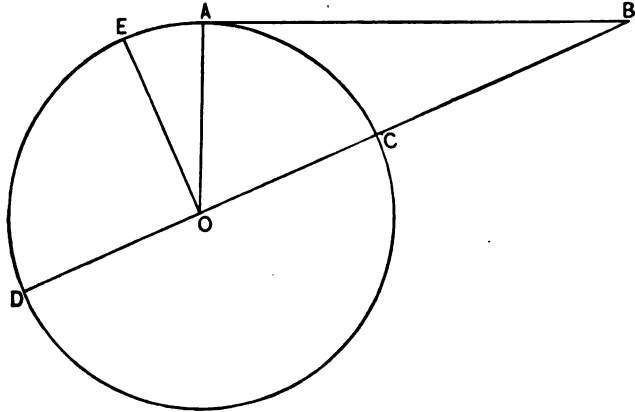


Fig. 30

Thus, in Fig. 30, the line A B is the tangent of the arc A C. The point of tangency is at A.

An angle at the center of a circle is measured by the arc intercepted by the sides of the angle. Hence the tangent A B of the arc A C is also the tangent of the angle A O B.

In the tables of circular functions the radius of the arc is unity, or, in common practice, we take it as one inch. The radius O A being 1", if we know the length of the line or tangent A B we can, by looking in a table of tangents, find the number of degrees in the angle A O B.

To find the  
Degrees in an  
Angle.

Thus, if A B is 2.25" long, we find the angle A O B is  $66^\circ$  very nearly. That is, having found that 2.2460 is the nearest number to 2.25 in the table of tangents at the end of this volume, we find the corresponding degrees of the angle in the column at the left hand of the table and the minutes to be added at the top of the column containing the 2.2460.

The table gives angles for every  $10'$ , which is sufficient for most purposes.

Now, if we have a right-angle triangle with an angle the same as  $O A B$ , but with  $O A$  two inches long, the line  $A B$  will also be twice as long as the tangent of angle  $A O B$ , as found in a table of tangents.

Let us take a triangle with the side  $O A = 5''$  long, and the side  $A B = 8''$  long; what is the number of degrees in the angle  $A O B$ ? Example to find the Degrees in an Angle.

Dividing  $8''$  by  $5$  we find what would be the length of  $A B$  if  $O A$  was only  $1''$  long. The quotient then would be the length of tangent when the *radius* is  $1''$  long, as in the table of tangents.  $8$  divided by  $5$  is  $1.6$ . The nearest tangent in the table is  $1.6003$  and the corresponding angle is  $58^\circ$ , which would be the angle  $A O B$  when  $A B$  is  $8''$  and the radius  $O A$  is  $5''$  very nearly. The difference in the angles for tangents  $1.6003$  and  $1.6$  could hardly be seen in practice. The side opposite the required acute angle corresponds to the tangent and the side adjacent corresponds to the radius. Hence the rule:

To find the tangent of either acute angle in a right-angle triangle: *Divide the side opposite the angle by the side adjacent the angle and the quotient will be the tangent of the angle.* To find the Tangent. This rule should be committed to memory. Having found the tangent of the angle, the angle can be taken from the table of tangents.

The complement of an angle is the remainder after subtracting the angle from  $90^\circ$ . Thus  $40^\circ$  is the complement of  $50^\circ$ . Complement of an Angle.

The *Cotangent* of an angle is the tangent of the complement of the angle. Cotangent. Thus, in Fig. 30, the line  $A B$  is the cotangent of  $A O E$ . In right-angle triangles either acute angle is the complement of the other acute angle. Hence, if we know one acute angle, by subtracting this angle from  $90^\circ$  we get the other acute angle. As the arc approaches  $90^\circ$  the tangent becomes longer, and at  $90^\circ$  it is infinitely long.

The sign of infinity is  $\infty$ .  $\text{Tangent } 90^\circ = \infty$ .



## CHAPTER II.

APPLICATION OF TABLE OF TANGENTS TO PROBLEMS OCCURRING IN  
MACHINE CONSTRUCTION—ANGLES OF BEVEL GEARS.

To avoid confusion we will illustrate one gear only. The same rules apply to all sizes of bevel gears. Fig. 31 is the outline of a pinion 4 P, 20 teeth, to mesh with a gear 28 teeth, shafts at right angles. For making sketch of bevel gears see Chapter IX, PART I.

In Fig. 31, the line  $O m' m$  is continued to the line  $a b$ . The angle  $c' O i$  that the cone pitch-line makes with the center line may be called the *center angle*.

Angle of  
Edge. Fig. 31.

The center angle  $c' O i$  is equal to the angle of edge  $c' i c$ .  $c' i$  is the side opposite the center angle  $c' O i$ , and  $c' O$  is the side adjacent the center angle.  $c' i = 2.5''$ ;  $c' O = 3.5''$ . Dividing  $2.5''$  by  $3.5''$  we obtain  $.71428''$  + as the tangent of  $c' O i$ . In the table we find  $.71329$  to be the nearest tangent, the corresponding angle being  $35^\circ 30'$ .  $35\frac{1}{2}^\circ$ , then, is the center angle  $c' O i$  and the angle of edge  $c' i n$ , very nearly.

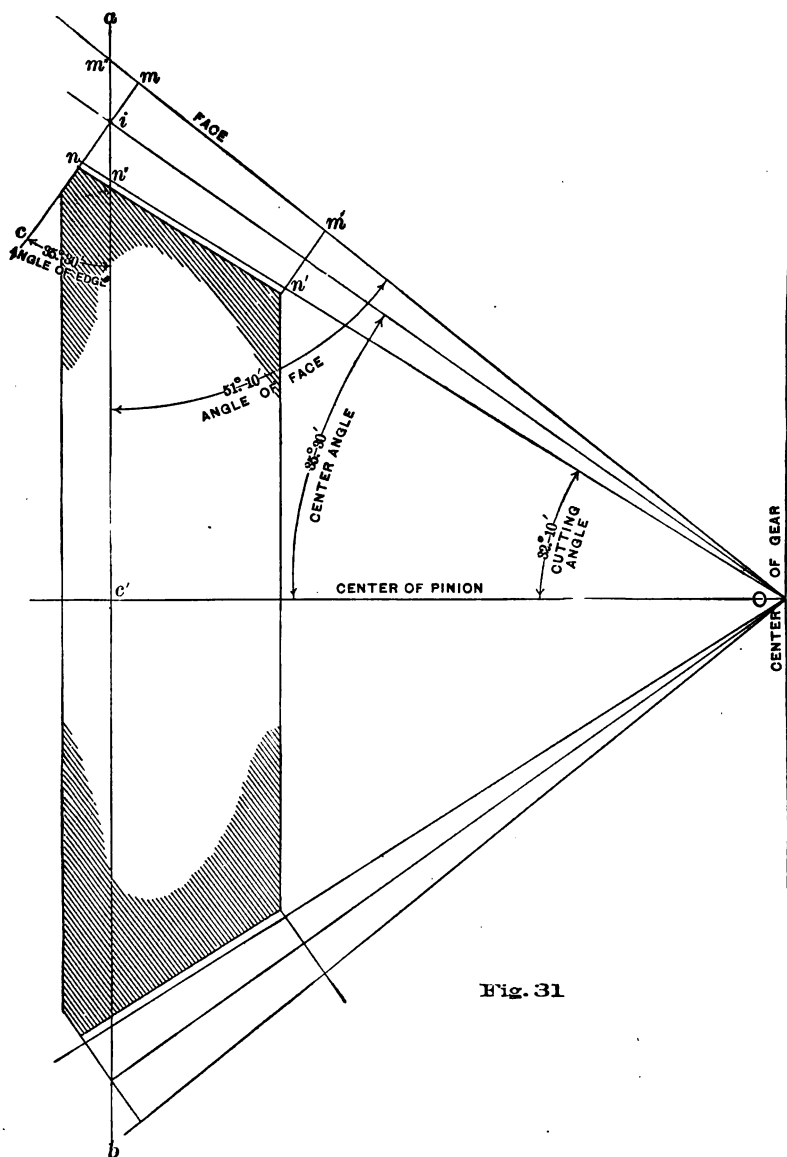
When the axes of bevel gears are at right angles the angle of edge of one gear is the complement of angle of edge of the other gear. Subtracting, then,  $35\frac{1}{2}^\circ$  from  $90^\circ$  we obtain  $54\frac{1}{2}^\circ$  as the angle of edge of gear 28 teeth, to mesh with gear 20 teeth, Fig. 31.

Angle of Face.

To obtain angle of face  $O m'' c'$ , the distance  $c' O$  becomes the side opposite and the distance  $m'' c'$  is the side adjacent.

The distance  $c' O$  is  $3.5''$ , the radius of the 28 tooth bevel gear. The distance  $c' m''$  is by measurement  $2.82''$ .

Dividing  $3.5$  by  $2.82$  we obtain  $1.2411$  for tangent of angle of face  $O m'' c'$ . The nearest tangent in the table is  $1.2422$  and the corresponding angle is  $51^\circ 10'$ .



To obtain cutting angle  $c' O n''$  we divide the distance  $c' n''$  by  $c' O$ . By measurement  $c' n''$  is 2.2". Dividing 2.2 by 3.5 we obtain .62857 for tangent of cutting angle. The nearest corresponding angle in the table is  $32^\circ 10'$ .

To lay out an Angle by the Tangent. Example, Fig. 32.

By a table of tangents, angles can be laid out upon sheet zinc, etc. This is often an advantage, as it is not convenient to lay protractor flat down so as to mark angles up to a sharp point. If we could lay off the length of a line *exactly* we could take tangents direct from table and obtain angle at once. It, however, is generally better to multiply the tangent by 5 or 10 and make an enlarged triangle. If, then, there is a slight error in laying off length of lines it will not make so much difference with the angle.

Let it be required to lay off an angle of  $14^\circ 30'$ . By the table we find the tangent to be .25861. Multiplying .25861 by 5 we obtain, in the enlarged triangle, 1.29305" as the length of side opposite the angle  $14^\circ 30'$ . As we have made the side opposite five times as large, we must make the side *adjacent* five times as large, in order to keep angle the same. Hence, Fig. 32, draw the line  $A B$  5" long; perpendicular to this line at  $A$  draw the line  $A O$  1.293" long; now draw the line  $O B$ , and the angle  $A B O$  will be  $14^\circ 30'$ .

If special accuracy is required, the tangent can be multiplied by 10; the line  $A O$  will then be 2.586" long and the line  $A B$  10" long. Remembering that the acute angles of a right-angle triangle are the complements of each other, we subtract  $14^\circ 30'$  from  $90'$  and obtain  $75^\circ 30'$  as the angle of  $A O B$ .

The reader will remember these angles as occurring in PART I., Chapter IV., and obtained in a different way. A semicircle upon the line  $O B$  touching the extremities  $O$  and  $B$  will just touch the right angle at  $A$ , and the line  $O B$  is four times as long as  $O A$ .

Let it be required to turn a piece 4" long, 1" diameter at small end, with a taper of  $10^\circ$  one side with the other; what will be the diameter of the piece at the large end?

A section, Fig. 33, through the axis of this piece is

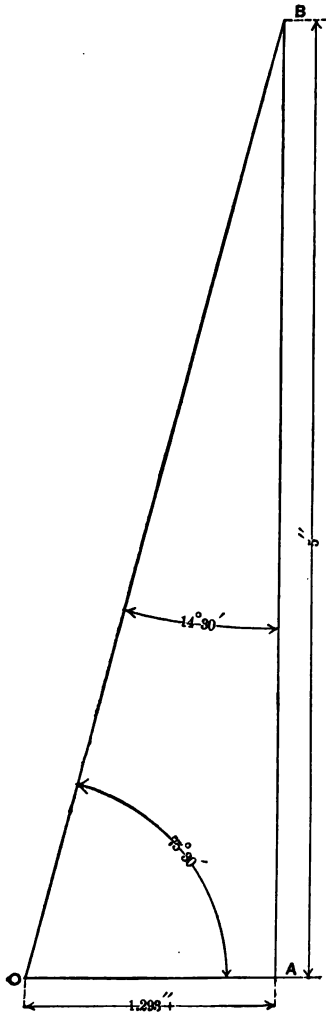


Fig. 32.

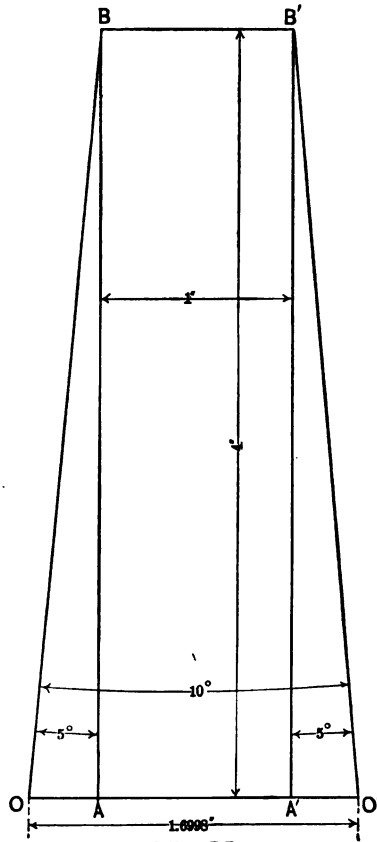


Fig. 33

To calculate the same as if we added two right-angle triangles, O  
 Diameter of a Tapering A B and O' A' B', to a straight piece A' A B B', 1"  
 piece. Fig. 33.

wide and 4" long, the acute angles B and B' being  $5^\circ$ , thus making the sides O B and O' B'  $10^\circ$  with each other.

The tangent of  $5^\circ$  is .08748, which, multiplied by 4", gives .34992" as the length of each line, A O and A' O', to be added to 1" at the large end. Taking twice .34992" and adding to 1" we obtain 1.69984" as the diameter of large end.

These chapters, I. and II., must be thoroughly studied before taking up the next chapters. If once the memory becomes confused as to the tangent and sine of an angle, it will take much longer to get righted than it will to first carefully learn to recognize the tangent of an angle at *once*.

If one knows what the tangent *is*, he can better tell the functions that are *not* tangents.

# CHAPTER III.

## SINE—COSINE AND SECANT: SOME OF THEIR APPLICATIONS IN MACHINE CONSTRUCTION.

*The Sine* of an arc is the line drawn from one extremity of the arc to the diameter passing through the other extremity, the line being perpendicular to the diameter.

Another definition is: The sine of an arc is the distance of one extremity of the arc from the diameter, through the other extremity.

The sine of an angle is the sine of the arc that <sup>Sine of Arc</sup> and Angle. measures the angle.

In Fig. 34, A C is the sine of the arc B C, and of the angle B O C. It will be seen that the sine is always inside of the arc, and can never be longer than

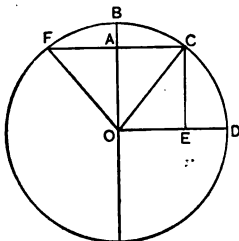


Fig. 34

the radius. As the arc approaches  $90^\circ$ , the sine comes nearer to the radius, and at  $90^\circ$  the sine is equal to 1, or is the radius itself. From the definition of a sine, the *side* A C, opposite the angle A O C, in *any* right-angle triangle, is the sine of the angle A O C, when O C is the *radius* of the arc.

Hence the rule: *In any right-angle triangle, the side opposite either acute angle, divided by the hypotenuse, is equal to the sine of the angle.* <sup>To find the Sine.</sup>

The quotient thus obtained is the length of side opposite the angle when the hypotenuse or radius is unity. The rule should be carefully committed to memory.

Chord of an Arc. *A Chord* is a straight line joining the extremities of an arc, and is twice as long as the sine of half the angle measured by the arc. Thus, in Fig. 34, the chord F C is twice as long as the sine A C.

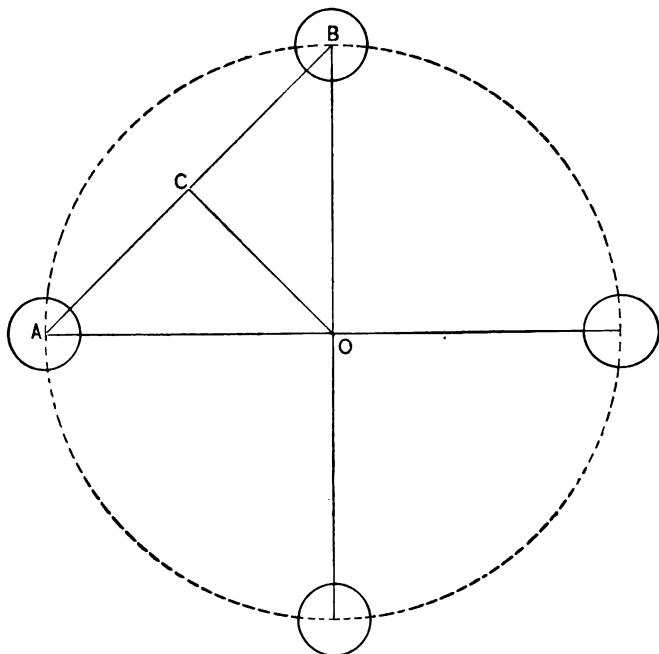


Fig. 35

Let there be four holes equidistant about a circle 3" in diameter—Fig. 35; what is the shortest distance between two holes? This shortest distance is the chord A B, which is twice the sine of the angle C O B. The angle A O B is one-quarter of the circle, and C O B is one-eighth of the circle.  $360^\circ$ , divided by  $8=45^\circ$ , the angle C O B. The sine of  $45^\circ$  is .70710, which, multiplied by 1.5", gives length C B in the circle, 3" in diameter, as 1.06065". Twice this length is the required distance A B=2.1213"

Example to find the Chord.

Polygon.

When a cylindrical piece is to be cut into any number of sides, the foregoing operation can be applied to obtain the width of one side. A plane figure bounded by straight lines is called a polygon.

When the outside diameter and the number of sides of a regular polygon are given, to find the length of one of the sides: *Divide  $360^\circ$  by twice the number of sides ; multiply the sine of the quotient by the outer diameter, and the product will be the length of one of the sides.* To find the length of Side.

Multiplying by the diameter is the same as multiplying by the radius, and that product again by 2.

*The Cosine of an angle is the sine of the complement of the angle.* Cosine.

In Fig. 34,  $\angle COD$  is the complement of the angle  $\angle AOC$ ; the line  $CE$  is the sine of  $\angle COD$ , and hence is the cosine of  $\angle AOC$ . The line  $OA$  is equal to  $CE$ . It is quite as well to remember the cosine as the part of the radius, from the center that is cut off by the sine. Thus the sine  $AC$  of the angle  $\angle AOC$  cuts off the cosine  $OA$ . The line  $OA$  may be called the cosine because it is equal to the cosine  $CE$ .

In any right-angle triangle, the side adjacent either acute angle corresponds to the cosine when the hypotenuse is the radius of the arc that measures the angle; hence: *Divide the side adjacent the acute angle by the hypotenuse, and the quotient will be the cosine of the angle.* To find the Cosine.

When a cylindrical piece is cut into a polygon of any number of sides, a table of cosines can be used to get the diameter across the sides. Length of sides of Polygon.

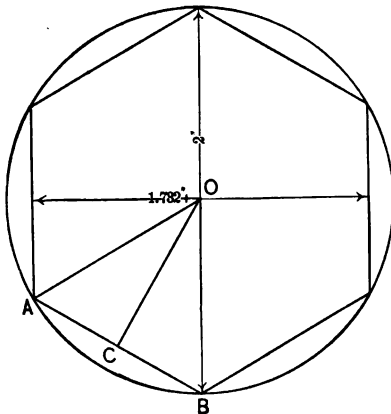


Fig. 36



Let a cylinder, 2" diameter, Fig. 36, be cut six-sided; what is the diameter across the sides?

The angle A O B, at the center, occupied by one of these sides, is one-sixth of the circle,  $=60^\circ$ . The cosine of one-half this angle,  $30^\circ$ , is the line C O; twice this line is the diameter across the sides. The cosine of  $30^\circ$  is .86602, which, multiplied by 2, gives 1.73204" as the diameter across the sides.

Of course, if the radius is other than unity, the cosine should be multiplied by the radius, and the product again by 2, in order to get diameter across the sides; or what is the same thing, multiply the cosine by the whole diameter or the diameter across the corners.

Rule for Diameter across sides of a Polygon.

The rule for obtaining the diameter across sides of regular polygon, when the diameter across corners is given, will then be: *Multiply the cosine of  $360^\circ$  divided by twice the number of sides, by the diameter across corners, and the product will be the diameter across sides.*

Look at the right-hand column for degrees of the cosine, and at bottom of page for minutes to add to the degrees.

Secant.

*The Secant of an arc is a straight line drawn from the center through one end of an arc, and terminated by a tangent drawn from the other end of the arc.*

Thus, in Fig. 37, the line O B is the secant of the angle C O B.

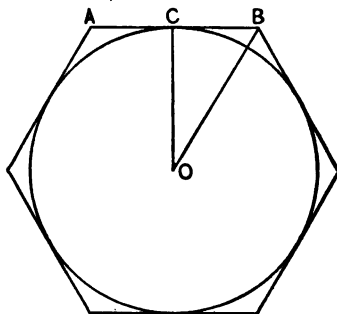


Fig. 37

To find the Secant.

*In any right-angle triangle, divide the hypotenuse by the side adjacent either acute angle, and the quotient will be the secant of that angle.*

That is, if we divide the distance  $OB$  by  $OC$ , in the right-angle triangle  $COB$ , the quotient will be the secant of the angle  $COB$ .

The secant cannot be less than the radius; it increases as the angle increases, and at  $90^\circ$  the secant is infinity  $=\infty$ .

A six-sided piece is to be  $1\frac{1}{4}$ " across the sides; how large must a blank be turned before cutting the sides? To find the Diameter across corners of a Polygon. Dividing  $360^\circ$  by twice the number of sides, we have  $30^\circ$ , which is the angle  $COB$ . The secant of  $30^\circ$  is 1.1547.

The radius of the six-sided piece is .75".

Multiplying the secant 1.1547 by .75", we obtain the length of radius of the blank  $OB$ ; multiplying again by 2, we obtain the diameter 1.732" +.

Hence, in a regular polygon, when the diameter across sides and the number of sides are given, to find diameter across corners: *Multiply the secant of  $360^\circ$  divided by twice the number of sides, by the diameter across sides, and the product will be the diameter across corners.*

It will be seen that the side taken as a divisor has been in each case the side corresponding to the radius of the arc that subtends the angle.

## CHAPTER IV.

# **FARTHER APPLICATION OF CIRCULAR FUNCTIONS—WHOLE DIAMETER OF BEVEL GEAR BLANKS—FACE ANGLE OF BEVEL GEAR BLANKS.**

The largest pitch diameter,  $kj$ , of a bevel gear, as in Fig. 38, is known the same as the pitch diameter of any spur gear. Now, if we know the distance  $bo$  or its equal  $aq$ , we can obtain the whole diameter of bevel gear blank by adding twice the distance  $bo$  to the largest pitch diameter.

Diameter In-  
crement. Fig.  
38.

Twice the distance  $bo$ , or what is the same thing, the sum of  $aq$  and  $bo$  is called the *diameter increment*, because it is the amount by which we increase the largest pitch diameter to obtain the whole or outside diameter of bevel gear blanks. The distance  $bo$  can be calculated without measuring the diagram.

The angle  $boj$  is equal to the angle of edge.

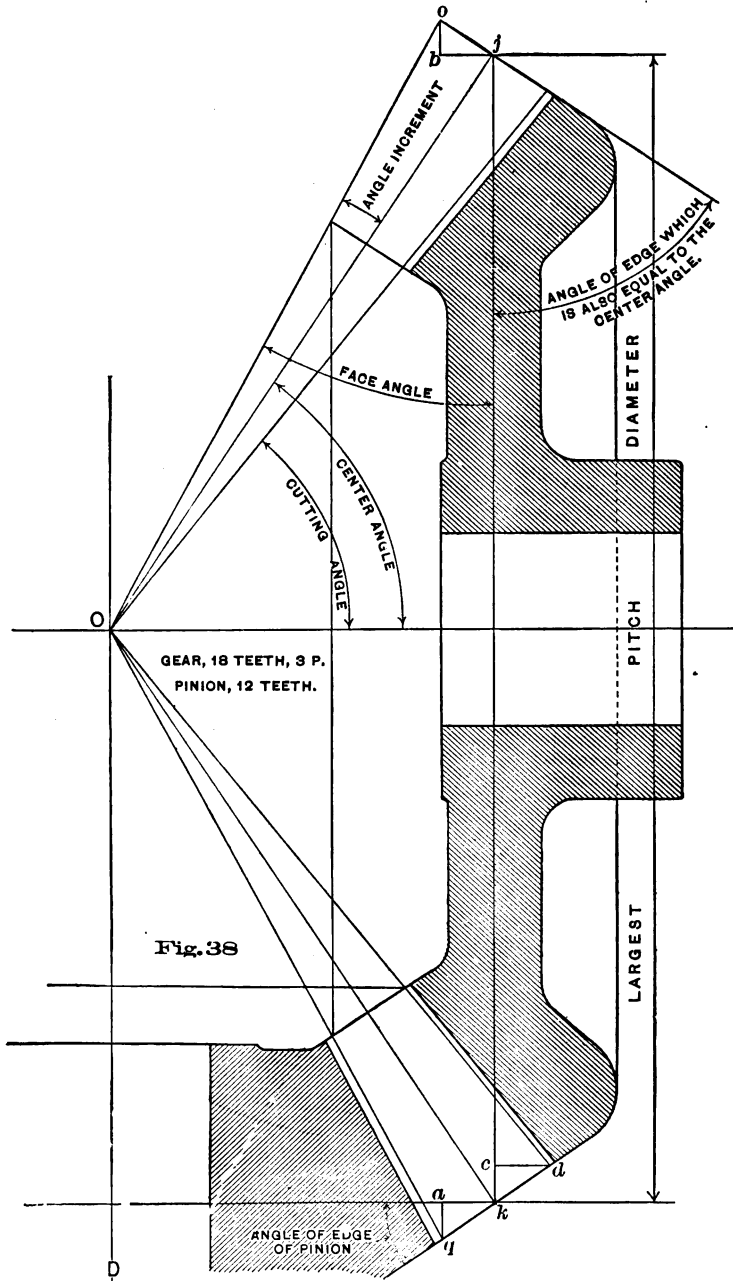
The angle of edge, it will be remembered, is the angle formed by outer edge of blank or ends of teeth with the end of hub or a plane perpendicular to the axis of gear.

The distance  $bo$  is equal to the cosine of angle of edge, multiplied by the distance  $jo$ . The distance  $jo$  is the addendum, as in previous chapters ( $=s$ ).

Hence the rule for obtaining the diameter increment of any bevel gear: *Multiply the cosine of angle of edge by the working depth of teeth ( $D'$ ), and the product will be the diameter increment.*

By the method given in Chapter II. we find the angle of edge of gear (Fig. 38) is  $56^\circ 20'$ . The cosine of  $56^\circ 20'$  is .55436, which, multiplied by  $\frac{3}{8}$ ", or the depth of the 3 P gear, gives the diameter increment of the bevel gear 18 teeth, 3 P meshing with pinion of 12

Outside Diam-  
eter.



teeth.  $\frac{3}{4}$  of .55436 = .369" + (or .37", nearly). Adding the diameter increment, .37", to the largest pitch diameter of gear, 6", we have 6.37" as the outside diameter.

In the same manner, the distance  $cd$  is half the diameter increment of the pinion. The angle  $cdk$  is equal to the center angle of pinion, and when axes are at right angles is the complement of center angle of gear. The center angle of pinion is  $33^\circ 40'$ . The cosine, multiplied by the working depth, gives .555" for diameter increment of pinion, and we have 4.555" for outside diameter of pinion.

In turning bevel gear blanks, it is sufficiently accurate to make the diameter to the nearest hundredth of an inch.

Angle Increment.

The small angle  $oOj$  is called the *angle increment*. When shafts are at right angles the face angle of one gear is equal to the center angle of the other gear, minus the angle increment.

Thus the angle of face of gear (Fig. 38) is less than the center angle  $DOk$ , or its equal  $Ojk$  by the angle  $oOj$ . That is, subtracting  $oOj$  from  $Ojk$ , the remainder will be the angle of face of gear.

Subtracting the angle increment from the center angle of gear, the remainder will be the cutting angle.

The angle increment can be obtained by dividing  $oj$ , the side opposite, by  $Oj$ , the side adjacent, thus finding the tangent as usual.

The length of cone-pitch line from the common center,  $O$  to  $j$ , can be found, without measuring diagram, by multiplying the secant of angle  $Ojk$ , or the center angle of pinion, by the radius of largest pitch diameter of gear.

The secant of angle  $Ojk$ ,  $33^\circ 40'$ , is 1.2015, which, multiplied by 3", the radius of gear, gives 3.6045" as the length of line  $Oj$ .

Dividing  $oj$  by  $Oj$ , we have for tangent .0924, and for angle increment  $5^\circ 20'$ .

The angle increment can also be obtained by the following rule:

*Divide the sine of center angle by half the number of teeth, and the quotient will be the tangent of increment angle.*

Subtracting the angle increment from center angles of gear and pinion, we have respectively:

Cutting angle of gear,  $51^{\circ}$ .

Cutting angle of pinion,  $28^{\circ} 20'$ .

Remembering that when the shafts are at right angles, the face angle of a gear is equal to the cutting angle of its mate (Chapter X.), we have:

Face angle of gear,  $28^{\circ} 20'$ .

Face angle of pinion,  $51^{\circ}$ .

It will be seen that both the whole diameter and the angles of bevel gears can be obtained without making a diagram. Mr. George B. Grant has made a table of different pairs of gears from 1 to 1 up to 10 to 1, containing diameter increments, angle increments and center angles, and has published it in the *American Machinist* of October 31, 1885. We have adopted the terms "diameter increment," "angle increment" and "center angle" from him. He uses the term "*back angle*" for what we have called *angle of edge*, only he measures the angle from the axis of the gear, instead of from the side of the gear or from the end of hub, as we have done; that is, his "back angle" is the complement of our *angle of edge*.

To lay out an  
Angle by the  
Sine.

In laying out angles, the following method may be preferred, as it does away with the necessity of making a right angle: Draw a circle, A B O (Fig. 39), ten

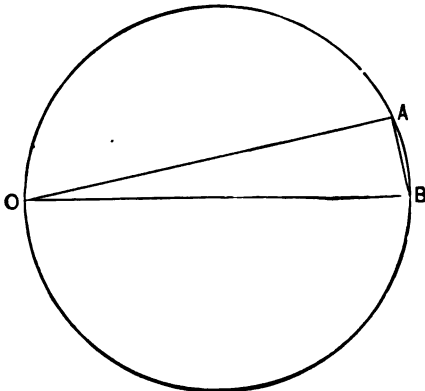


Fig. 39

inches in diameter. Set the dividers to ten times the sine of the required angle, and point off this distance in the circumference as at A B. From any point O in the circumference, draw the lines O A and O B. The angle A O B is the angle required. Thus, let the required angle be  $12^\circ$ . The sine of  $12^\circ$  is .20791, which, multiplied by 10, gives 2.0791", or  $2\frac{8}{100}$ " nearly, for the distance A B.

Any diameter of circle can be taken if we multiply the sine by the diameter, but 10" is very convenient, as all we have to do with the sine is to move the decimal point one place to the right.

If either of the lines, as O B, passes through the center, the angle O A B will be a right angle.

## CHAPTER V.

## SPIRAL GEARS—CALCULATIONS FOR PITCH OF SPIRALS.

When the teeth of a gear are cut, not in a straight <sup>Spiral Gear.</sup> path, like a spur gear, but in a helical or screw-like path, the gear is called, technically, a twisted or screw gear, but more generally among mechanics, a spiral gear. A distinction is sometimes made between a screw gear and a twisted gear. In twisted gears the pitch surfaces roll upon each other, exactly like spur gears, the axes being parallel, the same as in Fig. 1, Part I. In screw gears there is an end movement, or slipping of the pitch surfaces upon each other, the axes not being parallel. In screw gearing the action is analogous to a screw and nut, one gear driving another by the end movement of its tooth path. This is readily seen in the case of a worm and worm-wheel, when the axes are at right angles, as the movement of wheel is then wholly due to the end movement of worm thread. But, as we make the axes of gears more nearly parallel, they may still be screw gears, but the distinction is not so readily seen.

We can have two gears that are alike run together, with their axes at right angles, as at A B, Fig. 41.

The same gear may be used in a train of screw gears or in a train of twisted gears. Thus, B, as it relates to A, may be called a screw gear; but in connection with C, the same gear, B, may be called a twisted gear. These distinctions are not usually made, and we call all helical or screw-like gears made on the Universal Milling Machine *spiral gears*.

When two external spiral gears run together, with their axes parallel, the teeth of the gears must have opposite hand spirals.

Direction of  
Spiral with ref-  
erence to Axes.  
Fig. 41.



Thus, in Fig. 41 the gear B has right hand spiral teeth, and the gear C has left hand spiral teeth. When the axes of two spiral gears are at right angles, both gears must have the same hand spiral teeth. A and B, Fig. 41, have right hand spiral teeth. If both gears A and B had left hand spiral teeth, the relative direction in which they turn would be reversed.

#### Spiral Pitch.

The spiral pitch or pitch of spiral is the distance the spiral advances in one turn. Strictly, this is the *lead* of the spiral. A cylinder or gear cut with spiral grooves is merely a screw of coarse pitch or long lead; that is, a spiral is a coarse pitch screw, and a screw is a fine pitch spiral.

Since the introduction and extensive use of the Universal Milling Machine, it has become customary to call any screw cut in the milling machine a spiral. The spiral pitch is given as so many inches to one turn. Thus, a cylinder having a spiral groove that advances six inches to one turn, is said to have a six inch spiral.

In screws the pitch is often given as so many threads to one inch. Thus, a screw of  $\frac{1}{4}$ " lead is said to be 2 threads to the inch. The reciprocal expression is not much used with spirals. For example, it would not be convenient to speak of a spiral of 6" lead, as  $\frac{1}{6}$  threads to one inch.

The calculations for spirals are made from the functions of a right angle triangle.

Example,  
showing the nature  
of a Helix  
or Spiral.

Cut from paper a right angle triangle, one side about the right angle 6" long, and the other side about the right angle 2". Make a cylinder 6" in circumference. It will be remembered (Part I., Chapter II.) that the circumference of a cylinder, multiplied by .3183, equals the diameter— $6" \times .3183 = 1.9098"$ . Wrap the paper triangle around the cylinder, letting the 2" side be parallel to the axis, the 6" side perpendicular to the axis and reaching around the cylinder. The hypotenuse now forms a helix or screw-like line, called a spiral. Fasten the paper triangle thus wrapped around. See Fig. 42.

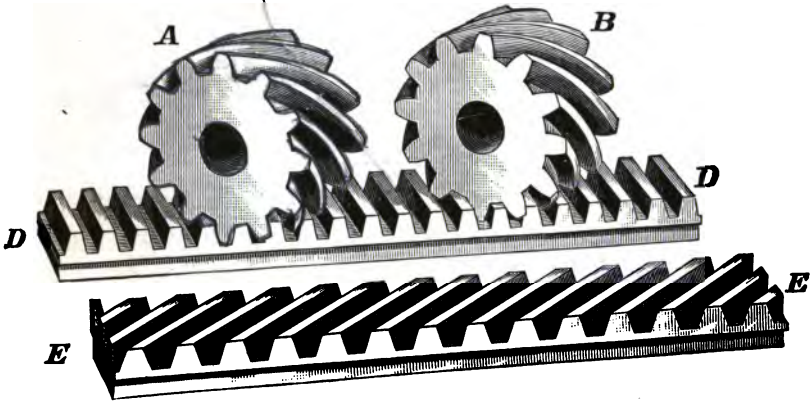


FIG. 40.—RACKS AND GEARS.

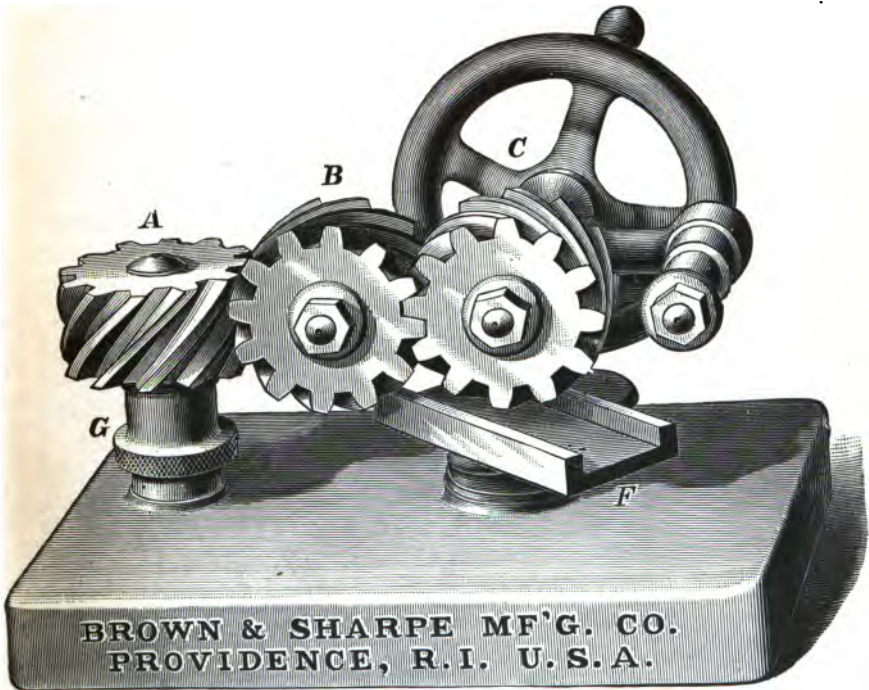


FIG. 41.—SPIRAL GEARING.

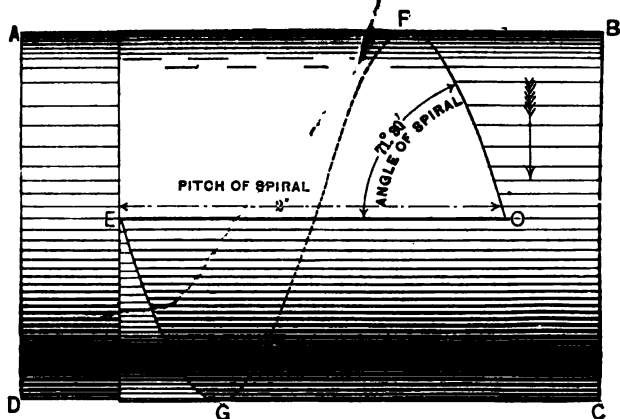


Fig. 42

If we now turn this cylinder A B C D in the direction of the arrow, the spiral will advance from O to E. This advance is the *pitch of the spiral*.

The angle E O F, which the spiral makes with the axis E O, is the *angle of the spiral*. This angle is found as in Chapter I. The circumference of the cylinder corresponds to the side *opposite the angle*. The pitch of the spiral corresponds to the side *adjacent the angle*. Hence the rule for getting angle of spiral:

Rules for calculating the parts of a Spiral.

*Divide the circumference of the cylinder or spiral by the number of inches of spiral to one turn, and the quotient will be the tangent of angle of spiral.*

When the angle of spiral and circumference are given, to find the pitch:

*Divide the circumference by the tangent of angle, and the quotient will be the pitch of the spiral.*

When the angle of spiral and the lead or pitch of spiral are given, to find the circumference:

*Multiply the tangent of angle by the pitch, and the product will be the circumference.*

Multiplying the circumference by .3183, we obtain the diameter of blank, as in Part I, Chapter II.

## CHAPTER VI.

EXAMPLES IN CALCULATION OF PITCH OF SPIRAL—ANGLE OF  
SPIRAL—CIRCUMFERENCE OF SPIRAL GEARS—  
A FEW HINTS ON CUTTING.

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It will be seen that the rules for calculating circumference of spiral gears, angle and pitch of spiral are the same as in Chapters I. and II. for tangent and angle of a right angle triangle. In Chapter V. the word "circumference" is substituted for "side opposite," and the words "pitch of spiral" are substituted for "side adjacent."

When two spiral gears are in mesh the angle of spiral should be the same in one gear as in the other, in order to have the shafts parallel and the teeth work properly together. When two gears both have right hand spiral teeth, or both have left hand spiral teeth, the angle of their shafts will be equal to the *sum* of the angles of their spirals. But when two gears have different hand spirals the angle of their shafts will be equal to the *difference* of their angles of spirals. Thus, in Fig. 41 the gears A and B both have right hand spirals. The angle of both spirals is  $45^\circ$ , their sum is  $90^\circ$ , or their axes are at right angles. But C has a left hand spiral of  $45^\circ$ . Hence, as the difference between angles of spirals of B and C is 0, their axes are parallel.

When the two gears have the same number of teeth the pitch of the spiral will be alike in both gears. But when one gear has more teeth than the other the pitch of spiral in the larger gear should be longer in the same ratio. Thus, if one gear has 50 teeth and the other gear has 25 teeth, the pitch of spiral in the 50 tooth wheel should be twice as long as that of the 25

Angles of Spirals with reference to Angle of Shafts.

Pitch in Spirals of different Diameters.

tooth wheel. Of course, the diameter of pitch circle should be twice as large in the 50 tooth as in the 25 tooth wheel.

In spirals where the angle is  $45^\circ$  the circumference is the same as the spiral pitch, because the tangent of  $45^\circ$  is 1.

Variation in  
Circumference  
to suit a Spiral.

Sometimes the circumference is varied to suit a pitch that can be cut on the machine and retain the angle required. This would apply to cutting rolls for making diamond-shaped impressions where the diameter of the roll is not a matter of importance.

When two gears are to run together in a given velocity ratio, it is well to first select spirals that the machine will cut of the same ratio, and calculate the numbers of teeth and angle to correspond. This will often save considerable time in figuring.

The calculations for spiral gears present no special difficulties, but sometimes a little ingenuity is required to make work conform to the machine and to such cutters as we may have in stock. It is a good plan to make a trial piece for each gear, and to cut a few teeth in each trial piece to test the setting of the machine.

Dummies or  
Trial Pieces.

These trial pieces are called "dummies." If the gears are likely to be duplicated, each dummy can be marked and kept for future setting of the machine. Stamp all the data on the dummies; it is better to spend a little time in marking dummies than a good deal of time hunting up, or trying to *remember*, old data.

Let it be required to make two spiral gears to run with a ratio of 4 to 1, the distance between centers to be  $3.125''$  ( $3\frac{1}{8}''$ ).

By rule given in Chapter XII., Part I., we find the diameters of pitch circles will be  $5''$  and  $1\frac{1}{4}''$ . Let us take a spiral of  $48''$  pitch for the large gear, and a spiral of  $12''$  pitch for the small gear. The circumference of the  $5''$  pitch circle is  $15.70796''$ . Dividing the circumference by the pitch of the spiral, we have  $\frac{15.70796}{48} = .32724''$  for tangent of angle of spiral. In the table the nearest angle to tangent,  $.32724''$ , is  $18^\circ 10'$ . As before stated, the angle of the teeth in the small gear will be the same as the angle of teeth or spiral in

the large gear. Now, this rule gives the angle at the pitch surface only. Upon looking at a small screw of coarse pitch, it will be seen that the angle at bottom of the thread is not so great as the angle at top of thread; that is, the thread at bottom is nearer parallel to the center line than that at the top.

A difference in Angles at top and bottom of Spiral Grooves.

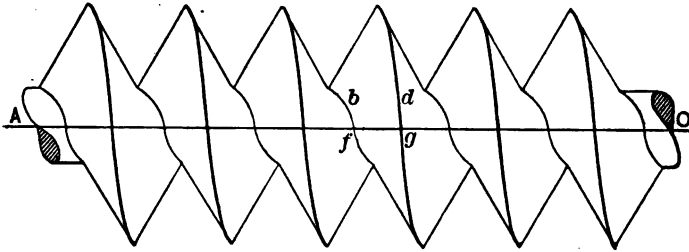


Fig. 43

This will be seen in Fig. 43, where A O is the center line;  $b f$  shows direction of bottom of the thread, and  $d g$  shows direction of top of thread. The angle  $A f b$  is less than the angle  $A g d$ . This difference of angle is due to the warped nature of a screw thread, and sometimes makes it necessary to change the angle for setting work from the figured angle, when a rotary disk cutter is used, to prevent the cutter from marring the groove as the teeth of cutter enter and leave. How much to change the angle can be seen by inspection when cutting the dummies. The change of angle will be more in a small gear of a given pitch than in a large gear of the same pitch.

A rotary disk cutter is generally preferable, because Disk-cutters.

it cuts faster and holds its shape better. Yet it is hardly practical to cut low numbered pinions with rotary disk cutters, because for some distance below the pitch line the spaces are so nearly parallel. A part of the difficulty can be removed by making the cutter as small as is consistent with strength. Still more of the trouble can be done away with by making a cutter on a shank, the center of the work and the center of shank cutter then being in the same plane. When using a shank cutter the center of the work is perpendicular to the center of the cutter, no adjustment for

Shank or End Cutter.

angle being made. Strictly, a shank mill does not reproduce its own shape in cutting a spiral groove. In using a shank cutter, more care is necessary to see that the work does not slip. It may be well to rough out with a disk cutter and finish with a shank cutter. There is not generally much difficulty in involute or single-curve spiral gears with disk cutters.

Example in  
calculation of  
Pitch of Spiral.

A cylinder 2" diameter is to have spiral grooves 20° with the center line of cylinder; what will be the pitch of spiral? The circumference is 6.2832". The tangent of 20° is .36397. Dividing the circumference by the tangent of angle, we obtain  $\frac{6.2832}{.36397} = 17.26'' +$  for pitch of spiral.

Before cutting into a blank it is well to make a slight trace of the spiral, with the cutter, after the machine is geared up, to see if the gears are properly arranged. Attention to this may avoid spoiling a blank.

The cutting of spiral gears develops some curious facts to one who may not have studied warped surfaces.

In the Universal Milling Machine we can cut a class of warped surfaces that will fit a straight edge in two directions. Thus, in Fig. 43, if it were possible to reduce the diameter of screw and then cut the thread clear down to the center line A O, the bottom of the thread would be a straight line running through the center or the line A O itself. The sides would still be straight as in the figure. If we should cut a spiral groove with a plain rotary disk cutter, having parallel sides, the shape of the grooves would have but little resemblance to that of the cutter. Taking advantage of this principle, we learned the fact that spiral gears can be planed with a rack tool.

Spiral Gears  
cut with Rack  
Tool.

The gears, Fig. 41, were planed. The tool was of the same shape as the spaces in the rack D D. All spiral gears of the same pitch could be planed with one tool.

The nature of this can be seen when we consider that straight rack teeth can mesh with spiral gears, as in Fig. 40.

We have succeeded in cutting small spiral gears with a long fly tool, cutting on one side only. The shape of this fly tool was like a common lathe side tool. In this case, of course, the gears had to be reversed in order to finish both sides of teeth.



## CHAPTER VII.

# NORMAL PITCH OF SPIRAL GEARS—CURVATURE OF PITCH SURFACE—FORM OF CUTTERS.

**Normal to a Curve.** A Normal to a curve is a line perpendicular to the tangent at the point of tangency.

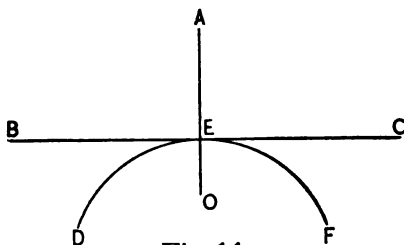


Fig. 44

In Fig. 44, the line  $BC$  is tangent to the arc  $DEF$ , and the line  $AE$ , being perpendicular to the tangent at  $E$ , the point of tangency, is a *normal* to the arc.

Fig. 45 is a representation of the pitch surface of a spiral gear.  $A'D'C'$  is the circular pitch, as in Part I.  $ADC$  is the same circular pitch seen upon the periphery of a wheel. Let  $AD$  be a tooth and  $DC$  a space. Now, to make this space  $DC$ , the path of cutting is along the dotted line  $ab$ . By mere inspection, we can see that the *shortest* distance between two teeth along the pitch surface is not the distance  $ADC$ .

Let the line  $AEB$  be perpendicular to the sides of teeth upon the pitch surface. A continuation of this line, perpendicular to all the teeth, is called the *Normal Helix*. The line  $AEB$ , reaching over a tooth and a space along the normal helix, is called the *Normal Pitch*.

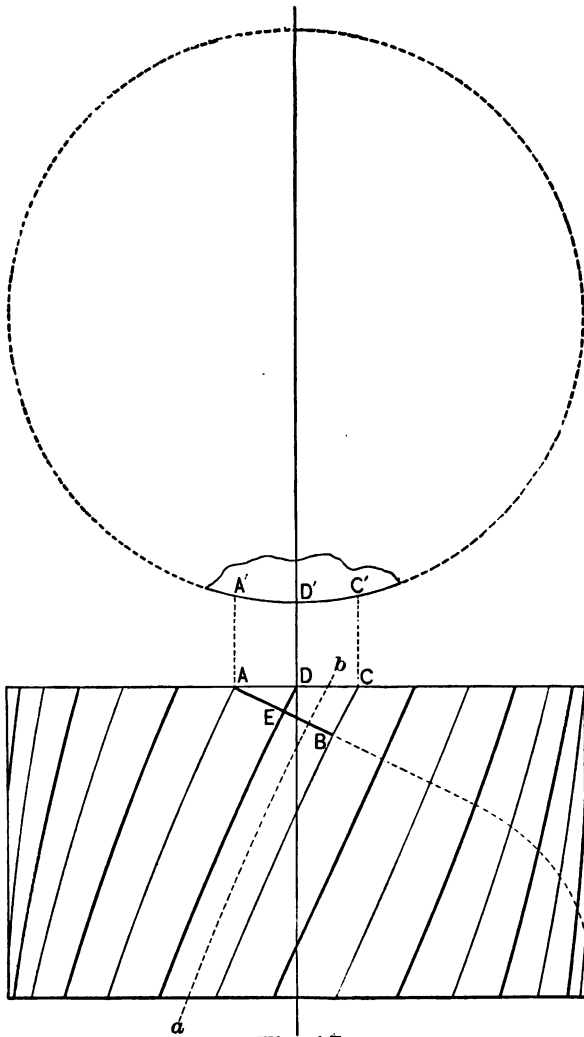


Fig. 45

**Normal Pitch.** The *Normal Pitch* of a spiral gear is then: *The shortest distance between the centers of two consecutive teeth measured along the pitch surface.*

In spur gears the normal pitch and circular pitch are alike. In the rack D D, Fig. 40, the linear pitch and normal pitch are alike.

**Cutter for Spiral Gears.**

From the foregoing it will be seen that, if we should cut the space D C with a cutter, the thickness of which at the pitch line is equal to one-half the *circular* pitch, as in spur wheels, the space would be too wide, and the teeth would be too thin. Hence, spiral gears should be cut with thinner cutters than spur gears of the same circular pitch.

**To find Normal Pitch.**

The angle C A B is equal to the angle of the spiral. The line A E B corresponds to the *cosine* of the angle C A B. Hence the rule: *Multiply the cosine of angle of spiral by the circular pitch, and the product will be the normal pitch.* One-half the normal pitch is the proper thickness of cutter at the pitch line.

The addendum and working depth of tooth should correspond to the *normal pitch*, and not to the circular pitch. Thus, if the normal pitch is 12 diametral, the addendum should be  $\frac{1}{2}$ ", the thickness .1309", and so on. The diameter of pitch circle of a spiral gear is calculated from the *diametral* pitch. Thus a gear of 30 teeth 10 P would be 3" pitch diameter.

But if the normal pitch is 12 diametral pitch, the blank will be  $3\frac{2}{3}$ " diameter instead of  $3\frac{5}{8}$ ".

**Normal Pitch varies.**

It is evident that the normal pitch varies with the angle of spiral. The cutter should be for the normal pitch. In designing spiral gears, it is well to first look over list of cutters on hand, and see if there are cutters to which the gears can be made to conform. This may avoid the necessity of getting a new cutter, or of changing both drawing and gears after they are under way. To do this, the problem is worked the reverse of the foregoing; that is:

**To make Angle of Spiral conform to Cutters given.**

First calculate to the next finer pitch cutter than would be required for the diametral pitch.

Let us take, for example, a gear 10 pitch and 30 teeth spiral. Let the next finer cutter be for 12 pitch

gears. The first thing is to find the angle that will make the normal pitch .2618", when the circular pitch is .3142". See table of tooth parts. This means (Fig. 45) that the line A D C will be .3142" when A E B is .2618". Dividing .2618" by .3142" (see Chapter III.), we obtain the cosine of the angle C A B, which is also the angle of the spiral,  $\frac{.2618}{.3142} = .833$ .

The same quotient comes by dividing 10 by 12.  $\frac{10}{12} = .833$ +. Looking in the table, we find the angle corresponding to the cosine .833 is 33° 30'. We now want to find the pitch of spiral that will give angle of 33½° on the pitch surface of the wheel, 3" diameter. Dividing the circumference by the tangent of angle, we obtain the pitch of spiral (see Chapter V.) The circumference is 9.4248". The tangent of 33° 30' is .66188,  $\frac{9.4248}{.66188} = 14.23$ ; and we have for our spiral 14.23" lead.

When the machine is not arranged for the exact pitch of spiral wanted, it is generally well enough to take the next nearest spiral. A half of an inch more or less in a spiral 10" pitch or more would hardly be noticed in angle of teeth. It is generally better to take the next longer spiral and cut enough deeper to bring center distances right. When two gears of the same size are in mesh with their axes parallel, a change of angle of teeth or spiral makes no difference in the correct meshing of the teeth.

But when gears of different size are in mesh, due regard must be had to the spirals being in pitch, proportional to their angular velocities (see Chapter VI.)

We come now to the curvature of cutters for spiral gears; that is, their shape as to whether a cutter is made to cut 12 teeth or 100 teeth. A cutter that is right, to cut a spur gear 3" diameter, may not be right for a spiral gear 3" diameter. To find the curvature of cutter, fit a templet to the blank along the line of the normal helix, as A E B, letting the templet reach over about two or three normal pitches. The curvature of this templet will be nearer a straight line than an arc of the addendum circle. Now find the diameter of a circle that will fit this templet, and consider this circle

When exact Pitch cannot be cut.

Spiral Gears of Different Sizes to Mesh.

Shape of Cutter.

as the addendum circle of a gear for which we are to select a cutter, reckoning the gear as of a pitch the same as the normal pitch.

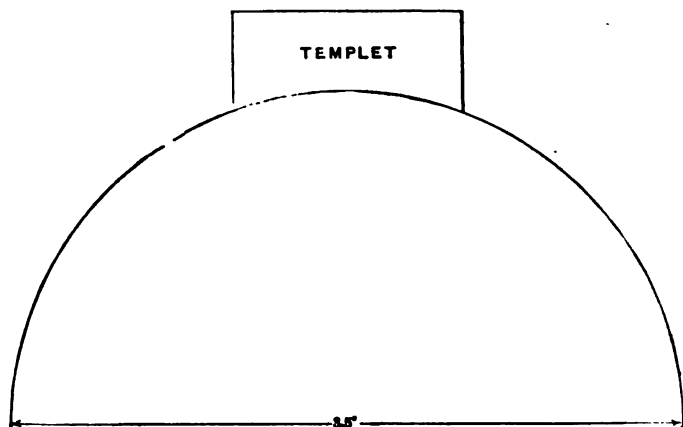


Fig. 46

Thus, in Fig. 46, suppose the templet fits a circle  $8\frac{1}{4}$ " diameter, if the normal pitch is 12 to inch, diametral, the cutter required is for 12 P and 40 teeth. The curvature of the templet will not be quite circular, but is sufficiently near for practical purposes. Strictly, a flat templet cannot be made to coincide with the normal helix for any distance whatever, but any greater refinement than we have suggested can hardly be carried out in a workshop.

The difference between normal pitch and linear or circular pitch is plainly seen in Figs. 40 and 41.

The rack D D, Fig. 40, is of regular form, the depth of teeth being  $\frac{1}{2}$  of the circular pitch, nearly (.6866 of the pitch, accurately). If a section of a tooth in either of the gears be made square across the tooth, that is a normal section, the depth of the tooth will have the same relation to the thickness of the tooth as in the rack just named.

But the teeth of spiral gears, looking at them upon the side of the gears, are thicker in proportion to their depth, as in Fig. 41. This difference is seen between

the teeth of the two racks D D and E E, Fig. 40. In the rack D D we have 20 teeth, while in the rack E E we have but 14 teeth; yet each rack will run with each of the spiral gears A, B or C, Fig. 41, but at different angles.

The teeth of one rack will accurately fit the teeth of the other rack face to face, but the sides of one rack will then be at an angle of  $45^{\circ}$  with the sides of the other rack. At F is a guide for holding a rack in mesh with a gear.

## CHAPTER VIII.

## SCREW GEARS AND SPIRAL GEARS—GENERAL REMARKS.

Working of  
Spiral Gears.

The working of spiral gears is generally smoother than spur gears. A tooth does not strike along its whole face or length at once. Tooth contact first takes place at one side of the gear, passes across the face and ceases at the other side of the gear. This action tends to cover defects in shape of teeth and the adjustment of centers.

Since the invention of machines for producing accurate epicycloidal and involute curves, it has not so often been found necessary to resort to spiral gears for smoothness of action. A greater range can be had in the adjustment of centers in spiral gears than in spur gears. The angle of the teeth should be enough, so that one pair of teeth will not part contact at one side of the gears until the next pair of teeth have met on the other side of the gears. When this is done the gears will be in mesh so long as the circumferences of their addendum circles intersect each other. This is sometimes necessary in roll gears.

End Pressure  
upon Shafts of  
Spiral Gears.

Relative to spur and bevel gears in Part I, Chapter XII, it was stated that all gears finally wore themselves out of shape and might become noisy. Spiral gears may be worn out of shape, but the smoothness of action can hardly be impaired so long as there are any teeth left. For every quantity of wear, of course, there will be an equal quantity of backlash, so that if gears have to be reversed the lost motion in spiral gears will be as much as in any gears, and may be more if there is end play of the shafts. In spiral gears there is end pressure upon the shafts, because of the screw-like action of the teeth. This end pressure is sometimes balanced by putting two gears upon each shaft, one of right and one of left hand spiral.

The same result is obtained in solid cast gears by making the pattern in two parts—one right and one left-hand spiral. Such gears are colloquially called “herring-bone gears.”

In an internal spiral gear and its pinion, the spirals of both wheels are either right-handed or left-handed. Such a combination would hardly be a mercantile product, although interesting as mechanical feat.

In screw or worm-gears the axes are generally at right angles, or nearly so. The distinctive features of screw gearing may be stated as follows:

The relative angular velocities do not depend upon the diameters of pitch-cylinders, as in Chapter I, Part I. Thus the worm in Chapter XI., Fig. 18, can be any diameter—one inch or ten inches—without affecting the velocity of the worm-wheel. The direction in which a worm-wheel turns depends upon whether the worm has a right-hand or left-hand thread. When angles of axes of worm and worm-wheel are oblique, there is a practical limit to the directional relation of the worm-wheel. The rotation of the worm-wheel is made by the end movement of the worm-thread.

The term worm and worm-wheel, or worm-gearing, is applied to cases where the worms are cut in a lathe.

If we let two cylinders touch each other, their axes be at right angles, the rotation of one cylinder will have no tendency to turn the other cylinder, as in Chapter I., Part I.

We can now see why worms and worm-wheels wear out faster than other gearing. The length of worm-thread, equal to more than the entire circumference of wheel during one turn of the wheel.

The angle of a worm-thread can be calculated the same as the angle of teeth of spiral gear.

Distinctive  
features of  
Screw Gearing.

Why Worm  
Wheels wear  
so fast.



## CHAPTER IX.

## CONTINUED FRACTIONS—SOME APPLICATIONS IN MACHINE CONSTRUCTION.

Definition of  
a Continued  
Fraction.

A continued fraction is one which has unity for its numerator, and for its denominator an entire number plus a fraction, which fraction has also unity for its numerator, and for its denominator an entire number plus a fraction, and thus in order.

The expression,  $\frac{1}{4 + \frac{1}{8 + \frac{1}{5}}}$

is called a continued frac-

Practical use  
of Continued  
Fractions.

tion. By the use of continued fractions, we are enabled to find a fraction expressed in smaller numbers, that, for practical purposes, may be sufficiently near in value to another fraction expressed in large numbers. If we were required to cut a worm that would mesh with a gear 4 diametral pitch (4 P.), in a lathe having 3 to 1-inch linear leading screw, we might, without continued fractions, have trouble in finding change gears, because the circular pitch corresponding to 4 diametral pitch is expressed in large numbers:  $4 P = \frac{78.54}{10000} P'$ .

This example will be considered farther on. For illustration, we will take a simpler example.

What fraction expressed in smaller numbers is nearest in value to  $\frac{29}{148}$ ? Dividing the numerator and the denominator of a fraction by the same number does not change the value of the fraction. Dividing both terms of  $\frac{29}{148}$  by 29, we have  $\frac{1}{5\frac{1}{8}}$  or, what is the same thing expressed as a continued fraction,  $\frac{1}{5 + \frac{1}{8}}$ . The continued fraction  $\frac{1}{5 + \frac{1}{8}}$  is exactly equal to  $\frac{29}{148}$ . If now, we reject the  $\frac{1}{8}$ , the fraction  $\frac{1}{5}$  will be larger

Example in  
Continued  
Fractions.

than  $\frac{1}{5 + \frac{1}{25}}$ , because the denominator has been diminished, 5 being less than  $5\frac{1}{25}$ .  $\frac{1}{5}$  is the nearest fraction to  $\frac{29}{146}$  expressed in smaller numbers than 29 for a numerator and 146 for a denominator. Reducing  $\frac{1}{5}$  and  $\frac{29}{146}$  to a common denominator, we have  $\frac{1}{5} = \frac{29}{145}$  and  $\frac{29}{146} = \frac{1}{5\frac{1}{25}}$ . Subtracting one from the other, we have  $\frac{1}{730}$ , which is the difference between  $\frac{1}{5}$  and  $\frac{29}{146}$ . Thus, in thinking of  $\frac{29}{146}$  as  $\frac{1}{5}$ , we have a pretty good idea of its value.

In the French metric system, a millimetre is equal to .03937 inch; what fraction in smaller terms expresses .03937" nearly? .03937, in a vulgar fraction, is  $\frac{3937}{100000}$ . Dividing both numerator and denominator by 3937, we have  $\frac{1}{25\frac{1575}{787}}$ . Rejecting from the denominator of the new fraction,  $\frac{1575}{787}$ , the fraction  $\frac{1}{25}$  gives us a pretty good idea of the value of .03937". If in the expression,  $\frac{1}{25 + \frac{1575}{787}}$ , we divide both terms of the fraction  $\frac{1575}{787}$  by 1575, the value will not be changed. Performing the division, we have  $\frac{1}{25 + \frac{1}{2 + \frac{787}{1575}}}$ .

We can now divide both terms of  $\frac{787}{1575}$  by 787, without changing its value, and then substitute the new fraction for  $\frac{787}{1575}$  in the continued fraction.

Dividing again, and substituting, we have:

$$\frac{1}{25 + \frac{1}{2 + \frac{1}{2 + \frac{1}{787}}}}$$

as the continued fraction that is exactly equal to .03937.

In performing the divisions, the work stands thus:

$$\begin{array}{r} 3937 \overline{) 100000} \quad (25 \\ \underline{7874} \\ 21260 \\ \underline{19685} \\ 1575 \quad 3937 \quad (2 \\ \underline{3150} \\ 787 \quad 1575 \quad (2 \\ \underline{1574} \\ 1 \quad 787 \quad (787 \\ \underline{787} \\ 0 \end{array}$$

That is, dividing the last divisor by the last remainder, as in finding the greatest common divisor. The

quotients become the denominators of the continued fraction, with unity for numerators. The denominators 25, 2, and so on, are called incomplete quotients, since they are only the entire parts of each quotient. The first expression in the continued fraction is  $\frac{1}{25}$ , or .04—a little larger than .03937. If, now, we take  $\frac{1}{25 + \frac{1}{2}}$ , we shall come still nearer .03937. The expression  $\frac{1}{25 + \frac{1}{2}}$  is merely stating that 1 is to be divided by  $25\frac{1}{2}$ . To divide, we first reduce  $25\frac{1}{2}$  to an improper fraction,  $\frac{51}{2}$ , and the expression becomes  $\frac{2}{51}$ , or one divided by  $\frac{51}{2}$ . To divide by a fraction, "Invert the divisor, and proceed as in multiplication." We then have  $\frac{2}{51}$  as the next nearest fraction to .03937.  $\frac{2}{51} = .0392+$ , which is smaller than .03937. To get still nearer, we take in the next part of the continued fraction, and have  $\frac{1}{25 + \frac{1}{2 + \frac{1}{2}}}$ .

We can bring the value of this expression into a fraction, with only one number for its numerator and one number for its denominator, by performing the operations indicated, step by step, commencing at the last part of the continued fraction. Thus,  $2 + \frac{1}{2}$ , or  $2\frac{1}{2}$ , is equal to  $\frac{5}{2}$ . Stopping here, the continued fraction would become  $\frac{1}{25 + \frac{1}{\frac{5}{2}}}$ .

Now,  $\frac{1}{\frac{5}{2}}$  equals  $\frac{2}{5}$ , and we have  $\frac{1}{25 + \frac{2}{5}}$ .  $25\frac{2}{5}$  equals  $1\frac{27}{5}$ ; substituting again, we have  $\frac{1}{1\frac{27}{5}}$ . Dividing 1 by  $1\frac{27}{5}$ , we have  $\frac{5}{127}$ .  $\frac{5}{127}$  is the nearest fraction to .03937, unless we reduce the whole continued fraction  $\frac{1}{25 + \frac{1}{2 + \frac{1}{25 + \frac{1}{2 + \frac{1}{2}}}}}$ , which would give us back the .03937 itself.

$\frac{5}{127} = .03937007$ , which is only  $\frac{7}{100000000}$  larger .03937. It is not often that an approximation will come so near as this.

Practical use  
of the foregoing  
Example.

This ratio, 5 to 127, is used in cutting millimeter thread screws. If the leading screw of the lathe was

1 to one inch, the change gears would have the ratio of 5 to 127; if 8 to one inch, the ratio would be 8 times as large, or 40 to 127; so that with leading screw 8 to inch, and change gears 40 and 127, we can cut millimeter threads near enough for practical purposes.

The foregoing operations are more tedious in description than in use. The steps have been carefully noted, so that the reason for each step could be seen from rules of common arithmetic, the operations being merely reducing complex fractions. The reductions,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{12}$ , etc., are called *convergents*, because they come nearer and nearer to the required .03937. The operations can be shortened as follows:

Let us find the fractions converging towards .7854", Example. the circular pitch of 4 diametral pitch,  $.7854 = \frac{7854}{10000}$ ; reducing to lowest terms, we have  $\frac{3927}{5000}$ . Applying the operation for the greatest common divisor:

$$\begin{array}{r}
 3927 \overline{) 5000} \quad (1 \\
 \underline{3927} \phantom{00} \\
 1073 \phantom{00} \quad (3 \\
 \underline{3219} \phantom{00} \\
 708 \phantom{00} \quad (1 \\
 \underline{708} \phantom{00} \\
 365 \phantom{00} \quad (1 \\
 \underline{365} \phantom{00} \\
 343 \phantom{00} \quad (1 \\
 \underline{343} \phantom{00} \\
 22 \phantom{00} \quad (15 \\
 \underline{22} \phantom{00} \\
 123 \phantom{00} \\
 \underline{110} \phantom{00} \\
 13 \phantom{00} \quad (22 \\
 \underline{13} \phantom{00} \\
 9 \phantom{00} \quad (13 \\
 \underline{9} \phantom{00} \\
 4 \phantom{00} \quad (9 \\
 \underline{4} \phantom{00} \\
 1 \phantom{00} \quad (4 \\
 \underline{1} \phantom{00} \\
 0
 \end{array}$$

Bringing the various incomplete quotients as denominators in a continued fraction as before, we have:

$$\begin{array}{c}
 1 \\
 1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{15 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4}}}}}}}}
 \end{array}$$

Now arrange each partial quotient in a line, thus:

$$\begin{array}{cccccccccc} 1 & 3 & 1 & 1 & 1 & 15 & 1 & 1 & 2 & 4 \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{15} & \frac{1}{13} & \frac{2}{55} & \frac{89}{1137} & \frac{2887}{7854} \end{array}$$

Now place under the first incomplete quotient the first reduction or convergent  $\frac{1}{1}$ , which, of course, is 1; put under the next partial quotient the next reduction or convergent  $\frac{1}{1+\frac{1}{2}}$  or  $\frac{1}{1\frac{1}{2}}$ , which becomes  $\frac{2}{3}$ .

1 is larger than .7854, and  $\frac{2}{3}$  is less than .7854.

Having made two reductions, as previously shown, we can shorten the operations by the following rule for next convergents: Multiply the numerator of the convergent just found by the denominator of the next term of the continued fraction, or the next incomplete quotient, and add to the product the numerator of the preceding convergent; the sum will be the numerator of the next convergent.

Proceed in the same way for the denominator. Continue until the last convergent is the original fraction. Under each incomplete quotient or denominator from the continued fraction arranged in line, will be seen the corresponding convergent or reduction. The convergent  $\frac{1}{1}$  is the one commonly used in cutting racks 4 P. The equivalent decimal to  $\frac{1}{1}$  is .7857+, being about  $\frac{3}{10000}$  large. In three settings for rack teeth, this error would amount to about .001".

For a worm, this corresponds to  $\frac{1}{1}$  threads to 1"; now, with a leading screw of lathe 3 to 1", we would want gears on the spindle and screw in a ratio of 33 to 14.

Hence, a gear on the spindle with 66 teeth, and a gear on the 3 thread screw of 28 teeth, would enable us to cut a worm to fit a 4 P gear.

## CHAPTER X.

## ANGLE OF PRESSURE.

In Fig. 47, let A be any flat disk lying upon a horizontal plane. Take any piece, B, with a square end,  $ab$ . Press against A with the piece B in the direction of the arrow.

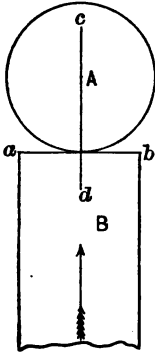


Fig. 47

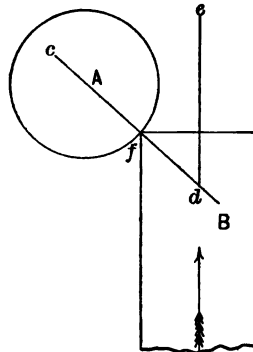


Fig. 48

It is evident A will tend to move directly ahead of B in the normal line  $cd$ . Now (Fig. 48) let the piece B, at one corner  $f$ , touch the piece A. Move the piece B along the line  $de$ , in the direction of the arrow.

It is evident that A will not now tend to move in the line  $de$ , but will tend to move in the direction of the normal  $cd$ . When one piece, not attached, presses against another, the tendency to move the second piece is in the direction of the normal, at the point of contact. This normal is called the *line of pressure*. The angle that this line makes with the path of the impelling piece, is called the *angle of pressure*.

In Part I., Chapter IV., the lines BA and BA' are called lines of pressure. This means that if the gear

drives the rack, the *tendency* to move the rack is not in the direction of pitch line of rack, but either in the direction B A or B A', as we turn the wheel to the left or to the right.

The same law holds if the rack is moved in the direction of the pitch line; the *tendency* to move the wheel is not directly tangent to the pitch circle, as if driven by a belt, but in the direction of the line of pressure. Of course the rack and wheel *do* move in the paths prescribed by their connections with the framework, the wheel turning about its axis and the rack moving along its ways. This pressure, not in a direct path of the moving piece, causes extra friction in all toothed gearing that cannot well be avoided.

Although this pressure works out by the diagram, as we have shown, yet, in the actual gears, it is not at all certain that they will follow the law as stated, because of the friction of teeth among themselves. If the driver in a train of gears has no bearing upon its tooth-flank, we apprehend there will be but little tendency to press the shafts apart.

Arc of Action.

The arc through which a wheel passes while one of its teeth is in contact is called the *arc of action*.

Base of System of Interchangeable Gears.

Until within a few years, the base of a system of double-curve interchangeable gears was 12 teeth. It is now 15 teeth in the best practice (see Chapter VII., Part I.)

The reason for this change was: the base, 15 teeth, gives less angle of pressure and longer arc of contact, and hence longer lifetime of gears.

## CHAPTER XI.

## INTERNAL GEARS—TABLES.

In Part I., Chapter VIII., it was stated that the space of an internal gear is the same as the tooth of a spur gear. This applies to involute or single-curve gears as well as to double-curve gears.

The sides of teeth in involute internal gears will be hollowing. It, however, has been customary to cut internal gears with spur gear-cutters, a No. 1 cutter generally being used. This makes the teeth sides convex. Special cutters should be made for coarse pitch double-curve gears. In designing internal gears, it is sometimes necessary to depart from the system with 15-tooth base, so as to have the pinion differ from the wheel by less than 15 teeth. The rules given in Part I., Chapters VII. and VIII., will apply in making gears on any base besides 15 teeth. If the base is low-numbered and the pinion is small, it may be necessary to resort to the method given at the end of Chapter VII., because the teeth may be too much rounded at the points by following the approximate rules. The base must be as small as the *difference* between the internal gear and its pinion. The base can be smaller if desired.

Special Cut-  
ters for coarse  
Pitch.

Base for In-  
ternal Gear  
Teeth.

Let it be required to make an internal gear, and pinion 24 and 18 teeth, 3 P. Here the base cannot be more than 6 teeth.

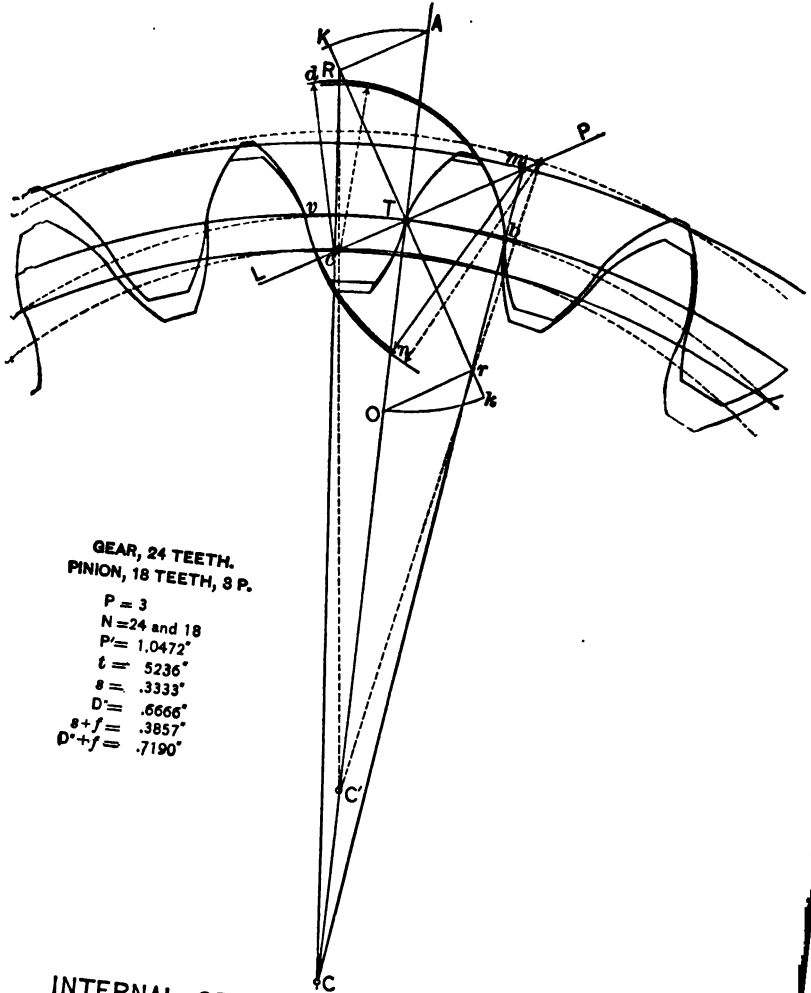
In Fig. 49 the base is 6 teeth. The arcs A K and O k, drawn about T, have a radius equal to the radius of the pitch circle of a 6-tooth gear, 3 P, instead of a 15-tooth gear, as in Chapter VIII., Part I.

The *outline* of teeth of both gears and pinion is made similar to the gear in Chapter VIII. The same

Description of  
Fig. 49.



Fig. 49



GEAR, 24 TEETH.  
PINION, 18 TEETH, 8 P.

$P = 3$   
 $N = 24$  and  $18$   
 $P' = 1.0472''$   
 $t = 5236''$   
 $s = .3333''$   
 $D' = .6666''$   
 $s + f = .3857''$   
 $D' + f = .7190''$

INTERNAL GEAR AND PINION IN MESH.

letters refer to similar parts. The clearance circle is, however, drawn on the *outside* for the internal gear. As before stated, the spaces of a spur wheel become the *teeth* of an internal wheel. The teeth of internal gears require but little for fillets at the roots; they are generally strong enough without fillets. The teeth of the pinion are also similar to the gear in Chapter VIII., substituting 6-tooth for 15-tooth base. To avoid confusion, it is well to make a complete sketch of one gear before making the other. The arc of action is longer in internal gears than in external gears. This property sometimes makes it necessary to give less fillets than in external gears.

In Fig. 49 the angle  $KTA$  is  $30^\circ$  instead of  $12^\circ$ , as in Fig. 12. This brings the line of pressure  $LP$  at an angle of  $60^\circ$  with the radius  $CT$ , instead of  $78^\circ$ . A system of spur gears could be made upon this 6-tooth base. These gears would interchange, but no gear of this 6-tooth system would mesh with a double-curve gear made upon the 15-tooth system in Part 1.

The "Willis" double-curve Odontograph is based upon radial flank gears of 12 teeth.

Base of Willis  
Odontograph.

## TABLE OF DECIMAL EQUIVALENTS

OF 8THS, 16THS, 32NDS AND 64THS OF AN INCH.

<b>8ths.</b>		$\frac{1}{32} = .28125$	$\frac{19}{64} = .296875$
$\frac{1}{8} = .125$	$\frac{1}{16} = .34375$	$\frac{1}{8} = .40625$	$\frac{21}{64} = .328125$
$\frac{2}{8} = .250$	$\frac{1}{8} = .46875$	$\frac{1}{4} = .53125$	$\frac{23}{64} = .359375$
$\frac{3}{8} = .375$	$\frac{1}{4} = .59375$	$\frac{3}{8} = .65625$	$\frac{25}{64} = .390625$
$\frac{4}{8} = .500$	$\frac{3}{8} = .71875$	$\frac{1}{2} = .78125$	$\frac{27}{64} = .421875$
$\frac{5}{8} = .625$	$\frac{1}{2} = .84375$	$\frac{5}{8} = .90625$	$\frac{29}{64} = .453125$
$\frac{6}{8} = .750$	$\frac{3}{4} = .96875$		$\frac{31}{64} = .484375$
$\frac{7}{8} = .875$			$\frac{33}{64} = .515625$
<b>16ths.</b>		<b>64ths.</b>	
$\frac{1}{16} = .0625$		$\frac{1}{64} = .015625$	$\frac{45}{64} = .703125$
$\frac{2}{16} = .1875$		$\frac{3}{64} = .046875$	$\frac{47}{64} = .734375$
$\frac{3}{16} = .3125$		$\frac{5}{64} = .078125$	$\frac{49}{64} = .765625$
$\frac{4}{16} = .4375$		$\frac{7}{64} = .109375$	$\frac{51}{64} = .796875$
$\frac{5}{16} = .5625$		$\frac{9}{64} = .140625$	$\frac{53}{64} = .828125$
$\frac{6}{16} = .6875$		$\frac{11}{64} = .171875$	$\frac{55}{64} = .859375$
$\frac{7}{16} = .8125$		$\frac{13}{64} = .203125$	$\frac{57}{64} = .890625$
$\frac{8}{16} = .9375$		$\frac{15}{64} = .234375$	$\frac{59}{64} = .921875$
<b>32nds.</b>		$\frac{17}{64} = .265625$	$\frac{61}{64} = .953125$
$\frac{1}{32} = .03125$			$\frac{63}{64} = .984375$
$\frac{3}{32} = .09375$			
$\frac{5}{32} = .15625$			
$\frac{7}{32} = .21875$			

## TABLE OF DECIMAL EQUIVALENTS

OF MILLIMETERS AND FRACTIONS OF MILLIMETERS.

mm. Inches.	mm. Inches.	mm. Inches.
$\frac{1}{16}$ = .00079	$\frac{25}{1000}$ = .02047	2 = .07874
$\frac{2}{16}$ = .00157	$\frac{26}{1000}$ = .02126	3 = .11811
$\frac{3}{16}$ = .00236	$\frac{27}{1000}$ = .02205	4 = .15748
$\frac{4}{16}$ = .00315	$\frac{28}{1000}$ = .02283	5 = .19685
$\frac{5}{16}$ = .00394	$\frac{29}{1000}$ = .02362	6 = .23622
$\frac{6}{16}$ = .00472	$\frac{30}{1000}$ = .02441	7 = .27559
$\frac{7}{16}$ = .00551	$\frac{31}{1000}$ = .02520	8 = .31496
$\frac{8}{16}$ = .00630	$\frac{32}{1000}$ = .02598	9 = .35433
$\frac{9}{16}$ = .00709	$\frac{33}{1000}$ = .02677	10 = .39370
$\frac{10}{16}$ = .00787	$\frac{34}{1000}$ = .02756	11 = .43307
$\frac{11}{16}$ = .00866	$\frac{35}{1000}$ = .02835	12 = .47244
$\frac{12}{16}$ = .00945	$\frac{36}{1000}$ = .02913	13 = .51181
$\frac{13}{16}$ = .01024	$\frac{37}{1000}$ = .02992	14 = .55118
$\frac{14}{16}$ = .01102	$\frac{38}{1000}$ = .03071	15 = .59055
$\frac{15}{16}$ = .01181	$\frac{39}{1000}$ = .03150	16 = .62992
$\frac{16}{16}$ = .01260	$\frac{40}{1000}$ = .03228	17 = .66929
$\frac{17}{16}$ = .01339	$\frac{41}{1000}$ = .03307	18 = .70866
$\frac{18}{16}$ = .01417	$\frac{42}{1000}$ = .03386	19 = .74803
$\frac{19}{16}$ = .01496	$\frac{43}{1000}$ = .03465	20 = .78740
$\frac{20}{16}$ = .01575	$\frac{44}{1000}$ = .03543	21 = .82677
$\frac{21}{16}$ = .01654	$\frac{45}{1000}$ = .03622	22 = .86614
$\frac{22}{16}$ = .01732	$\frac{46}{1000}$ = .03701	23 = .90551
$\frac{23}{16}$ = .01811	$\frac{47}{1000}$ = .03780	24 = .94488
$\frac{24}{16}$ = .01890	$\frac{48}{1000}$ = .03858	25 = .98425
$\frac{25}{16}$ = .01969	1 = .03937	26 = 1.02362

10 mm. = 1 Centimeter = 0.3937 inches.

10 cm. = 1 Decimeter = 3.937 "

10 dm. = 1 Meter = 39.37 "

25.4 mm. = 1 English Inch.

## NATURAL SINE.

Deg.	0'	10'	20'	30'	40'	50'	60'	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60'	50'	40'	30'	20'	10'	0'	Deg.

## NATURAL COSINE.

## NATURAL SINE.

Deg.	0'	10'	20'	30'	40'	50'	60'	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71984	.72185	.72380	.72587	.72787	.72986	.73185	43
47	.73185	.73383	.73580	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74703	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79689	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81243	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98723	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	.99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99993	.99993	.99990	1.0000	0
	00'	50'	40'	30'	20'	10'	0'	Deg.

## NATURAL COSINE.

## NATURAL TANGENT.

Deg.	0'	10'	20'	30'	40'	50'	60'	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02086	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03788	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64528	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	.80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83910	50
40	.83910	.84406	.84906	.85408	.85912	.86419	.86928	49
41	.86928	.87440	.87955	.88473	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92709	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97132	.97699	.98269	.98843	.99419	1.0000	45
	60'	50'	40'	30'	20'	10'	0'	Deg.

## NATURAL COTANGENT.

## NATURAL TANGENT.

Deg.	0'	10'	20'	30'	40'	50'	60	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1.2349	39
51	1.2349	1.2423	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3033	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3433	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5018	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0173	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0653	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.864	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60'	50'	40'	30'	20'	10'	0'	Deg.

## NATURAL COTANGENT.



## NATURAL SECANT.

Deg.	0'	10'	20'	30'	40'	50'	60'	
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0001	83
1	1.0001	1.0002	1.0002	1.0003	1.0004	1.0005	1.0005	88
2	1.0006	1.0007	1.0008	1.0009	1.0010	1.0012	1.0013	87
3	1.0013	1.0015	1.0016	1.0018	1.0020	1.0022	1.0024	86
4	1.0024	1.0026	1.0028	1.0030	1.0033	1.0035	1.0038	85
5	1.0038	1.0040	1.0043	1.0046	1.0049	1.0052	1.0055	84
6	1.0055	1.0058	1.0061	1.0064	1.0068	1.0071	1.0075	83
7	1.0075	1.0078	1.0082	1.0086	1.0090	1.0094	1.0098	82
8	1.0098	1.0102	1.0103	1.0111	1.0115	1.0120	1.0124	81
9	1.0124	1.0129	1.0134	1.0139	1.0144	1.0149	1.0154	80
10	1.0154	1.0159	1.0164	1.0170	1.0175	1.0181	1.0187	79
11	1.0187	1.0192	1.0198	1.0204	1.0210	1.0217	1.0223	78
12	1.0223	1.0229	1.0236	1.0242	1.0249	1.0256	1.0263	77
13	1.0263	1.0269	1.0277	1.0284	1.0291	1.0298	1.0303	76
14	1.0308	1.0313	1.0321	1.0329	1.0336	1.0344	1.0352	75
15	1.0352	1.0360	1.0369	1.0377	1.0385	1.0394	1.0402	74
16	1.0402	1.0411	1.0420	1.0429	1.0438	1.0447	1.0456	73
17	1.0456	1.0466	1.0475	1.0485	1.0494	1.0504	1.0514	72
18	1.0514	1.0524	1.0534	1.0544	1.0555	1.0565	1.0576	71
19	1.0576	1.0586	1.0597	1.0608	1.0619	1.0630	1.0641	70
20	1.0641	1.0653	1.0664	1.0676	1.0687	1.0699	1.0711	69
21	1.0711	1.0723	1.0735	1.0747	1.0760	1.0772	1.0785	68
22	1.0785	1.0798	1.0810	1.0823	1.0837	1.0850	1.0863	67
23	1.0863	1.0877	1.0890	1.0904	1.0918	1.0932	1.0946	66
24	1.0946	1.0960	1.0974	1.0989	1.1004	1.1018	1.1033	65
25	1.1033	1.1048	1.1063	1.1079	1.1094	1.1110	1.1126	64
26	1.1126	1.1141	1.1157	1.1174	1.1190	1.1206	1.1223	63
27	1.1223	1.1239	1.1256	1.1273	1.1290	1.1308	1.1325	62
28	1.1325	1.1343	1.1361	1.1378	1.1396	1.1415	1.1433	61
29	1.1433	1.1452	1.1470	1.1489	1.1508	1.1527	1.1547	60
30	1.1547	1.1566	1.1586	1.1605	1.1625	1.1646	1.1667	59
31	1.1666	1.1686	1.1707	1.1723	1.1749	1.1770	1.1791	58
32	1.1791	1.1813	1.1835	1.1856	1.1878	1.1901	1.1923	57
33	1.1923	1.1946	1.1969	1.1992	1.2015	1.2038	1.2062	56
34	1.2062	1.2085	1.2109	1.2134	1.2158	1.2182	1.2207	55
35	1.2207	1.2232	1.2257	1.2283	1.2308	1.2334	1.2360	54
36	1.2360	1.2386	1.2413	1.2440	1.2466	1.2494	1.2521	53
37	1.2521	1.2549	1.2576	1.2604	1.2632	1.2661	1.2690	52
38	1.2690	1.2719	1.2748	1.2777	1.2807	1.2837	1.2867	51
39	1.2867	1.2898	1.2928	1.2959	1.2990	1.3022	1.3054	50
40	1.3054	1.3086	1.3118	1.3150	1.3183	1.3216	1.3250	49
41	1.3250	1.3283	1.3317	1.3351	1.3383	1.3421	1.3456	48
42	1.3456	1.3491	1.3527	1.3563	1.3599	1.3636	1.3673	47
43	1.3673	1.3710	1.3748	1.3785	1.3824	1.3862	1.3901	46
44	1.3901	1.3940	1.3980	1.4020	1.4060	1.4101	1.4142	45
	00'	50'	40'	30'	20'	10'	0'	Deg.

## NATURAL COSECANT.

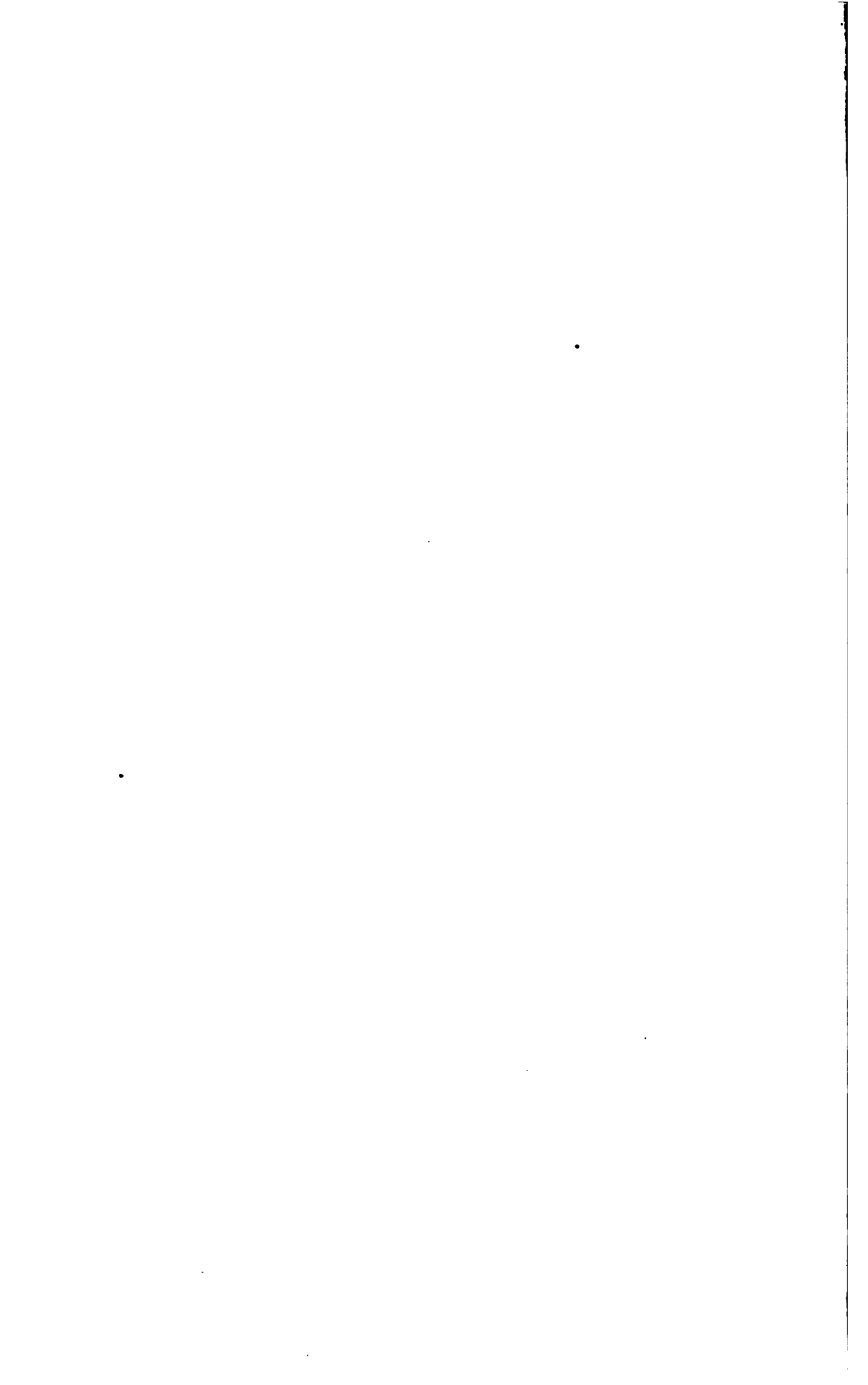
## NATURAL SECANT.

Deg.	0'	10'	20'	30'	40'	50'	60'	Deg.
45	1.4142	1.4188	1.4225	1.4267	1.4309	1.4352	1.4395	44
46	1.4395	1.4430	1.4463	1.4497	1.4532	1.4567	1.4602	43
47	1.4662	1.4708	1.4755	1.4801	1.4849	1.4896	1.4944	42
48	1.4944	1.4993	1.5042	1.5091	1.5141	1.5191	1.5242	41
49	1.5242	1.5293	1.5345	1.5397	1.5450	1.5503	1.5557	40
50	1.5557	1.5611	1.5666	1.5721	1.5777	1.5833	1.5890	39
51	1.5890	1.5947	1.6005	1.6063	1.6122	1.6183	1.6242	38
52	1.6242	1.6303	1.6364	1.6426	1.6489	1.6552	1.6616	37
53	1.6616	1.6680	1.6745	1.6811	1.6878	1.6945	1.7013	36
54	1.7013	1.7081	1.7150	1.7220	1.7291	1.7362	1.7434	35
55	1.7434	1.7507	1.7580	1.7655	1.7730	1.7806	1.7882	34
56	1.7882	1.7960	1.8038	1.8118	1.8198	1.8278	1.8360	33
57	1.8360	1.8443	1.8527	1.8611	1.8697	1.8783	1.8870	32
58	1.8870	1.8959	1.9048	1.9138	1.9230	1.9322	1.9416	31
59	1.9416	1.9510	1.9608	1.9702	1.9800	1.9899	2.0000	30
60	2.0000	2.0101	2.0203	2.0307	2.0412	2.0519	2.0626	29
61	2.0626	2.0735	2.0845	2.0957	2.1070	2.1184	2.1300	28
62	2.1300	2.1417	2.1538	2.1656	2.1778	2.1901	2.2026	27
63	2.2026	2.2153	2.2281	2.2411	2.2543	2.2676	2.2811	26
64	2.2811	2.2948	2.3087	2.3228	2.3370	2.3515	2.3662	25
65	2.3662	2.3810	2.3961	2.4114	2.4239	2.4426	2.4585	24
66	2.4585	2.4747	2.4911	2.5078	2.5247	2.5418	2.5593	23
67	2.5593	2.5769	2.5949	2.6131	2.6316	2.6503	2.6694	22
68	2.6694	2.6883	2.7085	2.7285	2.7488	2.7694	2.7904	21
69	2.7904	2.8117	2.8334	2.8554	2.8778	2.9006	2.9238	20
70	2.9238	2.9478	2.9713	2.9957	3.0205	3.0458	3.0715	19
71	3.0715	3.0977	3.1243	3.1515	3.1791	3.2073	3.2360	18
72	3.2360	3.2653	3.2951	3.3255	3.3564	3.3880	3.4203	17
73	3.4203	3.4531	3.4867	3.5209	3.5558	3.5915	3.6279	16
74	3.6279	3.6651	3.7031	3.7419	3.7816	3.8222	3.8637	15
75	3.8637	3.9061	3.9495	3.9939	4.0393	4.0859	4.1335	14
76	4.1335	4.1823	4.2323	4.2836	4.3362	4.3901	4.4454	13
77	4.4454	4.5021	4.5604	4.6202	4.6816	4.7448	4.8097	12
78	4.8097	4.8764	4.9451	5.0158	5.0886	5.1635	5.2408	11
79	5.2408	5.3204	5.4026	5.4874	5.5749	5.6653	5.7587	10
80	5.7587	5.8553	5.9553	6.0588	6.1660	6.2771	6.3924	9
81	6.3924	6.5120	6.6363	6.7654	6.8997	7.0396	7.1852	8
82	7.1852	7.3371	7.4957	7.6612	7.8344	8.0156	8.2055	7
83	8.2055	8.4046	8.6137	8.8336	9.0651	9.3091	9.5667	6
84	9.5667	9.8391	10.127	10.433	10.758	11.104	11.473	5
85	11.473	11.868	12.291	12.745	13.234	13.763	14.335	4
86	14.335	14.957	15.636	16.380	17.198	18.102	19.107	3
87	19.107	20.230	21.493	22.925	24.562	26.450	28.653	2
88	28.653	31.257	34.382	38.201	42.975	49.114	57.298	1
89	57.298	68.757	85.945	114.59	171.88	343.77	∞	0
	60'	50'	40'	30'	20'	10'	0'	Deg.

## NATURAL COSECANT.





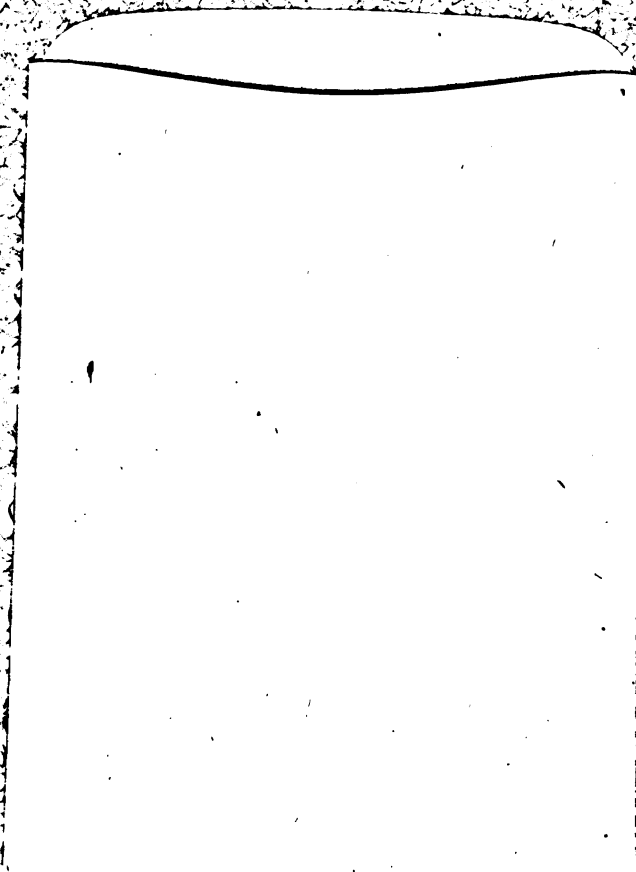


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